A Cosmological Model with Variable Deceleration Parameter

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We consider some exact solutions of Einstein field equations (EFEs) in the background of spatially homogeneous and totally anisotropic Bianchi type-I space-time. To get the deterministic models, the deceleration parameter q is assumed to be a simple linear function of Hubble's parameter H i.e., $q = -1 + \beta H$ (where β is a constant), which yields scale factor a as $a = e^{\frac{1}{\beta}\sqrt{2\beta t + k_3}}$ (where k_3 is a constant). The Universe model shows a transition from initial decelerating phase to present accelerating phase. It is seen that the model approaches isotropy at late times. We also discussed state-finder parameters, which predicts that the Universe in the model originates from Einstein's static era ($r \to \infty, s \to -\infty$) and marches towards ΛCDM model ($r = 1, s = 0$). We see that the model is in agreement with recent cosmological observations.

1. Introduction

Bianchi type-I cosmological models are interesting in the study because these models are homogeneous and anisotropic models and provide a better structure, both physically and geometrically, than the isotropic FRW (Friedmann-Robertson-Walker) models, and play a significant role in the description of early universe. These models (Bianchi-I) are the simplest anisotropic models, which have been suggested to give rise to an elliposoidality of the universe in spite of the inflation, which is one of the promising proposals to the solution of quadruple problem and can also be tested by the directional dependency of the red-shift luminosity relation of the SNe Ia observations [1-3]. There is significant evidence that the expansion of the observable universe is undergoing a late time acceleration. [4-11]. The Universe strikes a balance between simplicity and complexity that cosmologists are increasingly coming to understand. Recent observations of supernovae and large scale distribution of galaxies leave with the confounding discovery that the Universe seems to be dominated by a negative-gravity-like substance known as dark energy [12-21], which accelerates the rate at which the universe expands. The origin and nature of such an accelerating field poses a completely open question. Since by the 1990s, several developments in observational cosmology, especially discovery of the accelerating universe from distant supernovae in 1998, and also independent evidence from the cosmic microwave background (CMB) and large scale galaxy red-shift surveys have shown that the mass energy density of the Universe incorporates about 70 percent of dark energy, which is poorly understood at a fundamental level. The main required properties of dark energy are that it dilutes more slowly than matter as the Universe expands and that it clusters more weakly than matter. The cosmological constant is the simplest possible form of dark energy since it is constant in both space and time and this leads to the current standard model of cosmology known as ΛCDM model, which provides a good fit to many cosmological observations [22-24].

For any physically relevant model of the Universe, Hubble parameter H and deceleration parameter q are important observational quantities. The present value H_0 of Hubble parameter shows the present time rate of expansion, whereas the present value of deceleration parameter q_0 indicates that the expansion of current observable Universe is speeding up rather than slowing down as expected prior to SNe Ia observations. This hints that in a physically realistic model, the Universe should have decelerating expansion in the early phase of matter era to allow the formation of large scale structures followed by late time acceleration. The solution of Einstein field equations applying the law of the variation of Hubble parameter$H$, which yields a constant deceleration parameter, have been studied by several authors [25-29]. However, Singh [30] have proposed a form of Hubble parameter H as a function of scale factor a in the context of spatially homogeneous and anisotropic Bianchi type-I space-time in such away that the resulting deceleration parameter q has the

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desired property of signature flip. Type Ia supernovae (SNe Ia) observations and CMB anisotropies [31-33] have predicted decelerating expansion in the past followed by present accelerated expansion. Now for a universe, which has decelerating expansion in the past and an accelerating expansion at the present time, the deceleration parameter q must show signature flipping, which is also one of the motivations behind choosing a time dependent deceleration parameter. The time dependent q has also been studied by Pradhan et al. [34] in Bianchi type-I cosmological models within the framework of time dependent gravitational and cosmological constants and observes that the time dependence of q is reasonable for the present universe that gives an appropriate description of the evolution of the Universe. In the present study, we consider the deceleration parameter q to be a suitable linear function of Hubble's parameter H i.e., $q = 1 + \beta H$ which yields scale factor a as $a = e^{\frac{1}{\beta}\sqrt{2\beta t + k_3}}$ (where β and k_3 are constants). Consequences of the following three cases for the decay of Λ-term have been discussed.

Case I: $\Lambda \sim H^n$

Case I: $\Lambda \sim a^{-m}$

Case I: $\Lambda \sim \frac{\ddot{a}}{a}$

(where n and m are constants).

The phenomenological Λ decay scenarios have been studied by Chen and Wu [35], Carvalho et al. [36], Schutzhold [37], Vishwakarma [38], Arbab [39-40]. Furthermore, in order to solve cosmological problem, a variety of models with different decay laws for the variation of Λ-term have been reported during last two decades [41-52]. We organize this paper as follows: Sec. 2 contain metric and field equations, Sec. 3 involves solutions of field equations, Sec 4. is devoted to state-finder parameters and finally Sec. 5 gives concluding remarks.

2. Metric and Field Equations

We consider Bianchi type-I space-time in orthogonal form represented by

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2 \tag{1}$$

Where, $A(t)$, $B(t)$ and $C(t)$ are metric functions of time.

The Einstein field equations for time dependent Λ can be written as

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} + \Lambda g_{ij} \tag{2}$$

Where, R_{ij}, R, g_{ij} and Λ are Ricci tensor, Ricci scalar, metric tensor and cosmological constant, respectively.

Here T_{ij} is the energy-momentum tensor for perfect fluid given by

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij} \tag{3}$$

Where, ρ, p are energy and density, pressure, and v_i is the four velocity vector of the fluid given by the equation: $v_i v^i = -1$.

The equation of state parameter ω, which is considered as an important quantity in describing the dynamics of the Universe is the ratio of pressure p and energy density ρ, is given by

$$\omega = \frac{p}{\rho} \tag{4}$$

Where, $0 \leq \omega \leq 1$.

The field equations (2) for the metric in Eqn. (1) with matter distribution (Eqn. (3)) yield

$$-\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} = p - \Lambda \tag{5}$$

$$-\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{AC} = p - \Lambda \tag{6}$$

$$-\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} = p - \Lambda \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = \rho + \Lambda \tag{8}$$

Eqns. (5)-(7), yield

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{a^3} \tag{9}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{a^3} \tag{10}$$

Where, k_1 and k_2 are constants of integration.

We define average scale factor for Bianchi type-I space time as

$$a = (ABC)^{\frac{1}{3}} \tag{11}$$

From Eqns. (9)-(11), we obtain

$$A = m_1 a \, exp\left\{\frac{2k_1 + k_2}{3} \int \frac{dt}{a^3}\right\} \quad (12)$$

$$B = m_2 a \, exp\left\{\frac{k_2 - k_1}{3} \int \frac{dt}{a^3}\right\} \quad (13)$$

$$C = m_3 a \, exp\left\{\frac{-(k_1 + 2k_2)}{3} \int \frac{dt}{a^3}\right\} \quad (14)$$

Where, m_1, m_2 and m_3 are arbitrary constants of integration such that $m_1 m_2 m_3 = 1$.

We define Hubble's parameter H, expansion scalar θ, shear scalar σ, and deceleration parameter q as

$$H = \frac{\dot{a}}{a} \quad (15)$$

$$\theta = 3H = 3\frac{\dot{a}}{a} \quad (16)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \quad (17)$$

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 \quad (18)$$

Eqns. (5)-(8) can be rewritten in terms of H, σ and q as

$$p - \Lambda = 2H^2(2q - 1) - \sigma^2 \quad (19)$$

$$\rho + \Lambda = 3H^2 - \sigma^2 \quad (20)$$

3. Solutions of Field Equations

Eqns. (5)-(8) constitute a system of four equations in six unknowns $A, B, C, p, \rho,$ and Λ. Now to get the determinate solutions, we require two more conditions. Since for an expanding model of the Universe consistent with observations, one needs a specific deceleration parameter such that the model starts with decelerating expansion followed by late time acceleration.

So firstly we assume that the deceleration parameter q is a simple linear function of Hubble parameter$H$, that is

$$q = -1 + \beta H \quad (21)$$

Which, yields

$$a = e^{\frac{1}{\beta}\sqrt{2\beta t + k_3}} \quad (22)$$

Where, β and k_3 are constants.

From Eqns. (12)-(14) and (22), we find

$$A = m_1 e^{\frac{1}{\beta}\sqrt{2\beta t + k_3}} \, exp\left\{\frac{2k_1 + k_2}{3} \int \frac{dt}{e^{\frac{3}{\beta}\sqrt{2\beta t + k_3}}}\right\} \quad (23)$$

$$B = m_2 e^{\frac{1}{\beta}\sqrt{2\beta t + k_3}} \, exp\left\{\frac{k_2 - k_1}{3} \int \frac{dt}{e^{\frac{3}{\beta}\sqrt{2\beta t + k_3}}}\right\} \quad (24)$$

$$C = m_3 e^{\frac{1}{\beta}\sqrt{2\beta t + k_3}} \, exp\left\{\frac{-(k_1 + 2k_2)}{3} \int \frac{dt}{e^{\frac{3}{\beta}\sqrt{2\beta t + k_3}}}\right\} \quad (25)$$

From Eqns. (15)-(18) and Eqn. (22), Hubble parameter H, expansion scalar θ, shear scalar σ, and deceleration parameter q can be obtained as

$$H = \frac{1}{\sqrt{2\beta t + k_3}} \quad (26)$$

$$\theta = \frac{3}{\sqrt{2\beta t + k_3}} \quad (27)$$

$$\sigma = \frac{k}{\sqrt{3} e^{\frac{3}{\beta}\sqrt{2\beta t + k_3}}} \quad (28)$$

$$q = \frac{\beta}{\sqrt{2\beta t + k_3}} - 1 \quad (29)$$

The anisotropy parameter $\bar{A}$ for the model is given by

$$\bar{A} = \frac{2}{3}\frac{\sigma^2}{H^2} = \frac{2}{9}\frac{k^2(2\beta t + k_3)}{e^{\frac{6}{\beta}\sqrt{2\beta t + k_3}}} \quad (30)$$

Now, we discuss various cases resulting from different decay laws for the value of Λ.

Case I:
We consider

$$\Lambda = k_4 H^n \quad (31)$$

Where k_4 and n are constants.

From Eqns. (26) and (31), we obtain

$$\Lambda = \frac{k_4}{(2\beta t + k_3)^{\frac{n}{2}}} \quad (32)$$

The isotropic pressure p and energy density ρ, in view of Eqns. (19), (20) and (31), take the form as

$$p = \frac{4\beta}{(2\beta t + k_3)^{\frac{3}{2}}} - \frac{6}{2\beta t + k_3} - \frac{k^2}{3e^{\frac{6}{\beta}\sqrt{2\beta t + k_3}}} + \frac{k_4}{(2\beta t + k_3)^{\frac{n}{2}}} \quad (33)$$

$$\rho = \frac{3}{2\beta t + k_3} - \frac{k^2}{3e^{\frac{6}{\beta}\sqrt{2\beta t + k_3}}} - \frac{k_4}{(2\beta t + k_3)^{\frac{n}{2}}} \quad (34)$$

The density parameters for matter and vacuum $(\Omega_m, \Omega_\Lambda)$ are given by

$$\Omega_m = \frac{\rho}{3H^2} = 1 - \frac{k^2(2\beta t + k_3)}{9e^{\frac{6}{\beta}\sqrt{2\beta t + k_3}}} - \frac{k_4(2\beta t + k_3)}{3(2\beta t + k_3)^{\frac{n}{2}}} \quad (35)$$

$$\Omega_\Lambda = \frac{\Lambda}{3H^2} = \frac{k_4(2\beta t + k_3)}{(2\beta t + k_3)^{\frac{n}{2}}} \quad (36)$$

Eqns. (36) and (37) give the total density parameter (Ω) as

$$\Omega = \Omega_m + \Omega_\Lambda = 1 - \frac{k^2(2\beta t + k_3)}{9e^{\frac{6}{\beta}\sqrt{2\beta t + k_3}}} \quad (37)$$

We note that the initial time of the Universe in the model is $t = -\frac{k_3}{2\beta} = t'$ $(k_3 \geq 0, \beta > 0)$, which can be shifted to $t = 0$ by setting $k_3 = 0$. At this time, radius scale factor a is constant, which means that the Universe in the model is free from initial singularity. Also Hubble's parameter H, expansion scalar θ, isotropic pressure p and energy density ρ all are infinite at the initial time $(t = t')$, whereas the shear scalar σ is constant. Now as t increases scale factor a also increases while the physical parameters $H, \theta, p, \rho, \sigma$ all decrease and in the limit of large t, scale factor a becomes infinitely large but the parameters $H, \theta, p, \rho, \sigma$ converge to zero. This shows that the Universe in model starts from a non-singular state and expands exponentially with cosmic time t. We also see that the cosmological density Λ is infinite at the initial time $t = t'$ and $\Lambda \to 0$ as $t \to \infty$. This is in agreement recent astronomical observations. We also notice that the deceleration parameter q is positive for $t < \frac{\beta^2 - k_1}{2\beta}$, which indicates the decelerating phase of expansion and q is negative for $t > \frac{\beta^2 - k_1}{2\beta}$, which predicts the accelerating phase of expansion of the Universe in the present model. However, as $t \to \infty$, $q = -1$. This shows that the Universe in the model has transition from earlier decelerating phase to the present accelerating phase and shows the largest value of deceleration parameter q hence the fastest rate at which the Universe is undergoing expansion for large t. This future scenario of the Universe is also shown by the authors [53-54] and is in agreement with recent cosmological observations.

From Eqn. (30), we see that the anisotropy parameter $\bar{A}$ converges to zero as t tends to infinite. Therefore, the model approaches isotropy for large t. Equation (37) signifies that as $t \to \infty$, the term $\frac{k^2(2\beta t + k_3)}{9e^{\frac{6}{\beta}\sqrt{2\beta t + k_3}}}$ $(\beta > 0)$, approaches to zero hence $\Omega \to 1$, which is favored by recent observations till date.

The equation of state parameter ω is given by

$$\omega = -1 + \frac{4\beta(2\beta t + k_3)^{\frac{-3}{2}} - 3(2\beta t + k_3)^{-1} - \frac{2}{3}k^2 e^{\frac{-6}{\beta}\sqrt{2\beta t + k_3}}}{3(2\beta t + k_3)^{-1} - \frac{k^2}{3}e^{\frac{-6}{\beta}\sqrt{2\beta t + k_3}} - k_4(2\beta t + k_3)^{\frac{-n}{2}}} \quad (38)$$

From Eqn. (38), we see that the equation of state parameter ω is time dependent. Recent observational data limits the state parameter as $-1.67 < \omega < -0.62$ (SNe Ia data) while a combination of data from observations of SNe Ia, CMB anisotropy and galaxy clustering statistics gives the limit on ω as $-1.33 < \omega < -0.72$ [55-56]. In our model one can obtain the value of ω consistent with recent observations. As a matter of fact, if ω would be equal to -1 (standard ΛCDM cosmology), a little bit upper than -1 (the quintessence dark energy), less than -1 (phantom dark energy).

Case II:
Here we consider

$$\Lambda = k_5 a^{-m} \quad (39)$$

Where, k_5 and m are constants.

From Eqns. (22) and (39), we obtain

$$\Lambda = k_5 e^{\frac{-m}{\beta}\sqrt{2\beta t+k_3}} \tag{40}$$

For this, Eqns. (19), (20) and (40) give

$$p = 4\beta(2\beta t + k_3)^{\frac{-3}{2}} - 6(2\beta t + k_3)^{-1} - \frac{k^2}{3} e^{\frac{-6}{\beta}\sqrt{2\beta t+k_3}} + k_5 e^{\frac{-m}{\beta}\sqrt{2\beta t+k_3}} \tag{41}$$

$$\rho = 3(2\beta t + k_3)^{-1} - \frac{k^2}{3} e^{\frac{-6}{\beta}\sqrt{2\beta t+k_3}} + k_5 e^{\frac{-m}{\beta}\sqrt{2\beta t+k_3}} \tag{42}$$

Matter density parameter Ω_m and vacuum density parameter Ω_Λ takes the form as

$$\Omega_m = 1 - \frac{k^2}{9}(2\beta t + k_3) e^{\frac{-6}{\beta}\sqrt{2\beta t+k_3}} - \frac{k_5}{3}(2\beta t + k_3)^{\frac{-m}{\beta}\sqrt{2\beta t+k_3}} \tag{43}$$

$$\Omega_\Lambda = \frac{k_5}{3}(2\beta t + k_3)^{\frac{-m}{\beta}\sqrt{2\beta t+k_3}} \tag{44}$$

In addition Eqns. (43) and (44) lead to

$$\Omega = 1 - \frac{k^2}{9}(2\beta t + k_3) e^{\frac{-6}{\beta}\sqrt{2\beta t+k_3}} \tag{45}$$

In this case also, we see that the model has no initial singularity as the radius scale factor a is not zero at the initial time $t = t'$. The parameters H, θ, p, ρ all are infinite whereas σ is constant at the initial time $t = t'$. However in the limit of large t, scale factor a becomes infinitely large whereas the other parameters H, θ, p, ρ and σ drop to zero. This shows that the present model also starts from a singularity free state and continues to inflate till late times ($t \to \infty$). This scenario of the Universe is in agreement with recent astronomical observations. The cosmological term Λ is a genuine constant at $t = t'$ while $\Lambda \to 0$ as t tends to infinite, which is also observed by the researchers [57-58]. Here also, for some suitable epochs, one can obtain the decelerating and accelerating phases of expansion from the value of deceleration parameter q. We also observe the largest value of q and the fastest rate of expansion for late epochs. The same is in agreement with recent theoretical and experimental observations. Thus our model fits well within the limits of recent observations. We also see that anisotropy parameter $\bar{A}$ approaches to zero as t tends to infinite value, which shows that the Universe in the model isotropic for late times. Also, equation (45) shows that for late times (as $t \to \infty$), the total density parameter Ω approaches to 1, which agrees with recent observations.

From Eqn. (4), the equation of state parameter ω in this case can be determined as

$$\omega = -1 + \frac{4\beta(2\beta t + k_3)^{\frac{-3}{2}} - 3(2\beta t + k_3)^{-1} - \frac{2}{3}k^2 e^{\frac{-6}{\beta}\sqrt{2\beta t+k_3}}}{3(2\beta t + k_3)^{-1} - \frac{k^2}{3} e^{\frac{-6}{\beta}\sqrt{2\beta t+k_3}} - k_5 e^{\frac{-m}{\beta}\sqrt{2\beta t+k_3}}} \tag{46}$$

This shows that the state parameter ω is time dependent and its value lies in the range ($-1.67 < \omega < -0.62$).

Case III:
Lastly, we consider

$$\Lambda = k_6 \frac{\ddot{a}}{a} \tag{47}$$

Where, k_6 is an arbitrary constant.

From Eqns. (22) and (47), one can have

$$\Lambda = k_6 \left[\frac{1}{2\beta t + k_3} - \frac{\beta}{(2\beta t + k_3)^{\frac{3}{2}}}\right] \tag{48}$$

Eqns. (19), (20) and (48) give

$$p = \frac{4\beta}{(2\beta t + k_3)^{\frac{3}{2}}} - \frac{6}{2\beta t + k_3} - \frac{k^2}{3e^{\frac{6}{\beta}\sqrt{2\beta t+k_3}}} + k_6 \left[\frac{1}{2\beta t + k_3} - \frac{\beta}{(2\beta t + k_3)^{\frac{3}{2}}}\right] \tag{49}$$

$$\rho = \frac{3}{2\beta t + k_3} - \frac{k^2}{3e^{\frac{6}{\beta}\sqrt{2\beta t+k_3}}} - k_6 \left[\frac{1}{2\beta t + k_3} - \frac{\beta}{(2\beta t + k_3)^{\frac{3}{2}}}\right] \tag{50}$$

For this, the density parameters (Ω_m, Ω_Λ) are obtained as

$$\Omega_m = 1 - \frac{k^2}{9}(2\beta t + k_3) e^{\frac{-6}{\beta}\sqrt{2\beta t+k_3}} - k_6 \left[\frac{1}{3} - \frac{\beta}{(2\beta t + k_3)^{\frac{3}{2}}}\right] \tag{51}$$

$$\Omega_\Lambda = k_6\left[\frac{1}{3} - \frac{\beta}{(2\beta t + k_3)^{\frac{3}{2}}}\right] \qquad (52)$$

Adding equations (51) and (52), we obtain the total density parameter (Ω) as

$$\Omega = 1 - \frac{k^2(2\beta t + k_3)}{9e^{\frac{6}{\beta}\sqrt{2\beta t + k_3}}} \qquad (53)$$

Here, we observe that this case also presents the description of the Universe, which starts from a non-singular state and expands till late times. Scale factor a is constant while the parameters H, θ, p, ρ all are infinite at the initial time $t = t'$. The shear scalar σ is constant at $t = t'$. For infinitely large t, scale factor a becomes infinitely large whereas the parameters H, θ, p, ρ and σ vanish. Also, the cosmological term Λ is infinite at the initial time while $\Lambda \to 0$ as $t \to \infty$, which agrees with many observations in cosmology. The value of deceleration parameter q shows a transition from initial decelerating to present accelerating phase and gives the fastest rate at which the Universe is expanding for large times. This is the future scenario as is observed by many noted authors. We also see that the Universe approaches isotropy for large t because $\bar{A} \to 0$ as $t \to \infty$.

The equation of state parameter ω is obtained as

$$\omega = -1 + \frac{4\beta(2\beta t + k_3)^{\frac{-3}{2}} - 3(2\beta t + k_3)^{-1} - \frac{2}{3}k^2 e^{\frac{-6}{\beta}\sqrt{2\beta t + k_3}}}{3(2\beta t + k_3)^{-1} - \frac{k^2}{3}e^{\frac{-6}{\beta}\sqrt{2\beta t + k_3}} - k_6\left[\frac{1}{2\beta t + k_3} - \frac{\beta}{(2\beta t + k_3)^{\frac{3}{2}}}\right]} \qquad (54)$$

This signifies that ω depends on the cosmic time t and its value lies in the recent obsevational limits.

Also, from equation (53), we note that $\Omega \to 1$ as $t \to \infty$. This is in agreement with recent astronomical observations.

4. State-finder Parameters $\{r, s\}$

State-finder parameters $\{r, s\}$ play a significant role to describe the dynamics of the Universe in modern cosmological models. The dimensionless state-finder pair $\{r, s\}$ is introduced by Sahni et al. [59] and Alam et al. [60] and provides a perfect diagnosis of how much a dark energy model is close to ΛCDM dynamics. In recent times, state-finder parameters are frequently discussed for they provide a better idea about the geometry and become a powerful tool to distinguish between the dark energy models. The state-finder pair $\{r, s\}$ is defined as

$$r = \frac{\dddot{a}}{aH^2} = 1 + 3\frac{\dot{H}}{H} + \frac{\ddot{H}}{H^3} \qquad (55)$$

and

$$s = \frac{r-1}{3\left(q - \frac{1}{2}\right)} \qquad (56)$$

For a ΛCDM model, state-finder parameters have the value $\{r, s\}$=$\{1,0\}$. For Einstein era we have $\{r, s\}$=$\{\infty, -\infty\}$.

Now Eqns. (26), (29), (55) and (56) result into

$$r = 1 - \frac{3\beta(1-\beta)}{2\beta t + k_3} \qquad (57)$$

And

$$s = -\frac{3\beta(1-\beta)}{(2\beta t + k_3)\left[3\beta - 3\sqrt{2\beta t + k_3}\right]} \qquad (58)$$

We observe that as $t \to t'$, $\{r, s\} \to \{\infty, -\infty\}$ and as $t \to \infty$, $\{r, s\} \to \{1,0\}$, which depicts that the Universe in the model starts from Einstein static era and goes to ΛCDM model. This is in agreement with recent observations [61] and makes our model observationally acceptable.

5. Concluding Remarks

In the present paper, spatially homogeneous and totally anisotropic Bianchi type-I space-time is considered. We examined a cosmological scenario proposing a variation law in which the deceleration parameter q is assumed to be a simple linear function of Hubble's parameter H i.e., $q = -1 + \beta H$, which yields scale factor a as $a = e^{\frac{1}{\beta}\sqrt{2\beta t + k_3}}$ (where β, k_3 are constants). A class of solutions to Einstein field equations have been obtained by taking three phenomenological laws for the decay of cosmological term Λ as $\Lambda \sim H^n$, $\Lambda \sim a^{-m}$ and $\Lambda \sim \frac{\ddot{a}}{a}$ (where n, m are arbitrary constants). We have obtained cosmological models in which the Universe starts from a non-singular state and expands exponentially with cosmic time t till late times. The deceleration parameter q in the model is found to be time-dependent. It is seen that q shows a transition from initial decelerating phase to the present accelerating phase of expansion and supplies the largest value and the fastest rate at which the universe is expanding. Same is also observed by the researchers [62-63]. The cosmological term Λ approaches to zero as t tends to infinite also shown by recent observations [64-65]. We also observe that the equation of state parameter ω is a function of cosmic time t and its value in the model lies in the present observational limits. The anisotropy parameter $\bar{A}$ converges to zero as t tends to infinite, which signifies that the Universe model attains isotropy for large t. We also note that total density parameter Ω approaches to 1 as t goes to infinity. This is consistent with recent theoretical and experimental observations [66-69]. Finally, we noticed from the state-finder parameters that the model universe starts from Einstein static era $(r \rightarrow \infty, s \rightarrow -\infty)$ and goes to ΛCDM model $(r \rightarrow 1, s \rightarrow 0)$ [70]. Thus our model is in harmony with recent cosmological observations.

References

[1] D. C. Rodrigues, Phys. Rev. D **77**, 023534 (2008).

[2] Ö. Akarsu and C. B. Kilinc, Gen. Rel. Grav. **42**, 119 (2010).

[3] M. Watanabe, S. Kanno and J. Soda, Phys. Rev. Lett. **102**, 191302 (2009).

[4] S. Purlmutter et al., Astrophys. J. **483**, 565 (1997).

[5] S. Purlmutter et al., Nature **391**, 51 (1998).

[6] S. Purlmutter et al., Astrophys. J. **517**, 565 (1999).

[7] A. G. Riess et al., Astron. J. **116**, 1009 (1998).

[8] A. G. Riess et al., Astron. J. **607**, 665 (2004).

[9] S. W. Allen et al., Mon. Not. R. Astron. Soc. **353**, 457 (2004).

[10] J. A. S. Lima, Braz. J. Phys. **34**, 194 (2004).

[11] T. Padmanabhan, Phys. Rep. **380**, 235 (2003).

[12] F. Oliver, Julio C. Fabris Piattela and W. Zimdahl, JCAP **1105**, 029 (2011).

[13] I. Brevik and O. Gorbunova, Gen. Rel. Grav. **37**, 2039 (2005).

[14] J. Chen, S. Zhou and Y. Wang, Chin. Phys. Lett. **28**, 029801 (2011).

[15] A. Sheykhi and M. R. Setare, Int. J. Theor. Phys. **49**, 2777 (2010).

[16] H. Amirhashchi, Astrophys. Space Sci. **345**, 439 (2013).

[17] H. Amirhashchi, Astrophys. Space Sci. DoI: 10.1007/s10509-013-1675-z (2013b).

[18] ∅. Gron, Astrophys. Space. Sci. **173** 191 (1990).

[19] T. Padmanabhan and S. Chitre, Phys. Lett. A **120**, 433 (1987).

[20] J. D. Barrow, Phys. Lett. B **180**, 335 (1986).

[21] W. Zimdahl, Phys. Rev. D **53**, 5483 (1996).

[22] J. M. Overduin and F. I. Cooperstock, Phys. Rev. D **58**, 043506 (1998).

[23] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).

[24] K. Freese, New Astron. Rev. **49**, 103 (2005).

[25] M. S. Berman, Il Nuovo Cimento B **74**, 182 (1993).

[26] M. S. Berman and F. M. Gomide, Gen. Rel. Grav. **20**, 191 (1998).

[27] V. B. Johri and K. Desikan, Pramana-J. Phys. **42**, 473 (1994).

[28] G. P. Singh and K. Desikan, Pramana-J. Phys. **49**, 205 (1997).

[29] S. D. Maharaj and R. Naidoo, Astrophys. Space Sci. **208**, 261 (1993).

[30] J. P. Singh, Int. J. Theor. Phys. DoI 10.1007/ s10773-009-9880-6 (2009).

[31] M. R. Nolta et al. (WMAP Collaboration), Astrophys. J. Suppl. Ser. **180**, 296 (2009).

[32] K. Land and J. Magueijo, Phys. Rev. Lett. **95**, 071301 (2005).

[33] E. F. Bun and A. Buodon, Phys. Rev. D **78**, 123509 (2008).

[34] A. Pradhan, B. Saha and V. Rikhivitsky, arXiv: 1308.4842v2 [Physics-gen-ph] (2013).

[35] W. Chen and Y. S. Wu, Phys. Rev. D **41**, 695 (1990).

[36] J. C. Carvalho, J. A. S. Lima and I. Waga, Phys. Rev. D **46**, 2404 (1992).

[37] R. Schutzhold, Phys. Rev. Lett. **89**, 081302 (2002).

[38] R. G. Vishwakarma, Class. Quantum. Grav. **17**, 3833 (2000).

[39] A. I. Arbab, Gen. Rel. Grav. **29**, 61 (1997).

[40] A. I. Arbab, Gen. Rel. Grav. **30**, 1401 (1998).

[41] D. Pavon, Phys. Rev. D **43**, 375 (1991).

[42] J. A. S. Lima and J. M. F. Maia, Phys. Rev. D **49**, 5579 (1994).

[43] J. A. S. Lima and M. Traddon, Phys. Rev. D **53**, 4280 (1996).
[44] A. I. Arbab and A.-M.-M. Abdel-Rahaman, Phys. Rev. D **50**, 7725 (1994).
[45] R. G. Vishwakarma, Gen. Rel. Grav. **33**, 1973 (2001).
[46] J. V. Cunha and R. C. Santos, Int. J. Mod. Phys. D **13**, 1321 (2004).
[47] S. Carneiro and J. A. S. Lima, Int. J. Mod. Phys. A **20**, 2465 (2005).
[48] H. A. Borges and S. Carneiro, Gen Rel. Grav. **37**, 1385 (2005).
[49] R. K. Tiwari and Rameshwar Singh, Eur. Phys. J. Plus **129**, 253 (2014).
[50] R. K. Tiwari, Astrophys. Space Sci. **318**, 243 (2008).
[51] R. K. Tiwari and S. Kumar, Fizika B **19**, 211 (2010).
[52] R. K. Tiwari, Astrophys. Space. Sci. **321**, 147 (2009).
[53] S. Kumar and C. P. Singh, DoI 10.1007/s10714-010-1125-Y (2010).
[54] A. K. Yadav, Eur. Phys. J. Plus **129**, 194 (2014).
[55] M. Tegmark et al., Astrophys. J. **606**, 702 (2004).
[56] A. K. Yadav, F. Rahaman and S. Ray, arXiv: 1006.5412v2 [gr-qc] (2010).
[57] M. S. Berman, Int. J. Theor. Phys. **29**, 567 (1990).
[58] M. S. Berman and F. M. Gomoide, Gen. Rel. Grav. **22**, 625 (1990).
[59] V. Sahni et al., JETP Lett. **77**, 201 (2003).
[60] U. Alam et al., Mon. Not. R. Astron. Soc. **344**, 1057 (2003).
[61] Nasr Ahmad and Anirudh Pradhan, arXiv: 1303.3000v1 [Phys. Gen-ph] (2013).
[62] S. Kumar and A. K. Yadav, MPLA **26**, 647 (2011).
[63] S. Kumar and Ö. Akarsu, Eur. J. Phys. Plus **127**, 64 (2012).
[64] H. Amirhashchi, Phys. Lett. B **697**, 429 (2011).
[65] A. K. Yadav, Chin. Phys. Lett. **29**, 9801 (2012).
[66] C. L. Bennett et al., Astrophys. J. Suppl. **148**, 1 (2003).
[67] M. Tegmark at al. (SDSS Collabaration), Phys. Rev. D **69**, 103501 (2004).
[68] D. N. Spergel et al., Astrophys. J. Suppl. **148**, 175 (2003).
[69] P. de Bernardis et al., Nature **666**, 102 (2007).
[70] A. Pradhan, arXiv: 1211.1882v2 [Phys gen-ph] (2013).

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