Kantowski-Sachs Holographic Dark Energy In Brans-Dicke Theory of Gravitation

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In this paper, we have studied the anisotropic and homogeneous Kantowski-Sachs cosmological model filled with two minimally interacting fields - matter and holographic dark energy components in the frame work of Brans-Dicke (1961) scalar tensor theory of gravitation. To obtain a deterministic solution of the model we consider two conditions (i) shear scalar is proportional to expansion scalar (ii) scalar field is a function of average scale factor. Some important physical and geometrical properties are also discussed.

1. Introduction

Recent astrophysical data from distant Ia supernovae observations [2,3] show that the current Universe is not only expanding, but also it is accelerating due to some kind of negative-pressure form of matter known as dark energy [4,5]. The simplest candidate for dark energy is the cosmological constant [6], conventionally associated with the energy of the vacuum with constant energy density and pressure, and an equation of state $\omega = -1$. The present observational data favor an equation of state for the dark energy with parameter very close to that of the cosmological constant. The next simple model proposed for dark energy is the quintessence [7-9], a dynamical scalar field, which slowly rolls down in a flat enough potential. The equation of state for a spatially homogeneous quintessence scalar field satisfies $\omega = -1$ and therefore can produce accelerated expansion. This field is taken to be extremely light which is compatible with its homogeneity and avoids the problem with the initial conditions.

Recent studies of black holes and string theories may provide a new alternative to the solution of the dark energy problem, known as the holographic principle [10-13]. Another way to study dark energy arises from holographic principle that states that the number of degrees of freedom related directly to entropy scales with the enclosing area of the system. In that case, the total energy of the system with size L should not exceed the mass of the same black hole size. It means $L^3\rho_\Lambda = LM_p^2$, where ρ_Λ is the quantum zero-point energy density that comes from UV cutoff Λ, and M_p denotes the Planck mass. The largest L is required to saturate this inequality. Then, its holographic energy density is given by the following expression

$$\rho_\Lambda = \frac{3c^2 M_p^2}{L^2} \tag{1}$$

Where, c is free dimensionless parameter, which is commonly considered as a constant, but there is a possibility to consider non-constant c [14,15]. Based on cosmological state of the holographic principle, the holographic model of dark energy has been proposed and studied widely in the literature [16-24]. Recently, Adhav et al. [25] have discussed interacting dark matter and holographic dark energy in Bianchi type-*V* universe.

The Brans and Dicke [1] theory of gravitation is the well-known modified version of Einstein's theory. It is a scalar tensor theory in which the gravitational interaction is mediated by a scalar field as well as the tensor field g_{ij} of Einstein's theory. In this theory the scalar field has the dimension of the inverse of the gravitational constant.

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$$R_{ij}-\frac{1}{2}R\,g_{ij}=-8\pi\phi^{-1}T_{ij}-\omega\,\phi^{-2}\left(\phi_{,i}\,\phi_{,l}-\frac{1}{2}g_{ij}\,\phi_{,k}\,\phi^{,k}\right)-\phi^{-1}\left(\phi_{i;\,j}-g_{ij}\,\phi_{;k}^{;k}\right) \tag{2}$$

and

$$\phi_{;k}^{;k}=8\pi(3+2\omega)^{-1}T \tag{3}$$

Also, we have energy conservation equation as

$$T^{ij}{}_{;\,j}=0 \tag{4}$$

Where, T_{ij} is the stress energy tensor of the matter, ω is the dimensionless coupling constant and comma and semi-colon denote partial and covariant differentiation, respectively.

Several aspects of Brans-Dicke cosmology have been extensively investigated by many authors. Reddy [26], Adhav et al. [27], Rao and Vijaya Santhi [28-30], Naidu et al. [31] are some of the authors who have investigated several aspects of this theory. Pawar and Solanke [32] have discussed exact Kantowski-Sachs anisotropic dark energy cosmological models in the Brans-Dicke theory of gravitation. Recently, Kiran et al. [33] have studied holographic dark energy model in this theory.

Inspired by the above investigations and discussions, in this paper we study the Kantowski-Sachs holographic dark energy cosmological model in Brans-Dicke scalar tensor theory of gravitation. The paper is organized as follows. In Sec. 2, we discuss metric, energy momentum tensor and field equations. In Sec. 3, we have obtained solution of the field equations. Sec. 4 contains some important properties of the model. The last section devoted to conclusions of the obtained model.

2. Metric and Field Equations

We consider the spatially homogenous and anisotropic Kantowski–Sachs space-time in the form

$$ds^2=dt^2-A^2dr^2-B^2(d\theta^2+\sin^2\theta d\varphi^2) \tag{5}$$

Where, $A(t)$ and $B(t)$ are the functions of the cosmic time t only. The energy momentum tensor for the dark matter and holographic dark energy are respectively defined as

$$T_{ij}=\rho_m u_i u_j \tag{6}$$

$$\bar{T}_{ij}=(\rho_\lambda+p_\lambda)u_i u_j-p_\lambda g_{ij} \tag{7}$$

Here ρ_m, ρ_λ are the energy densities of dark matter and the holographic dark energy and p_λ is the pressure of the holographic dark energy.

In a co-moving coordinates system, from Eqns. (6) and (7), we get

$$T_1^1=T_2^2=T_3^3=0,\ T_4^4=\rho_m$$

and

$$\bar{T}_1^1=\bar{T}_2^2=\bar{T}_3^3=-p_\lambda,\ \bar{T}_4^4=\rho_\lambda \tag{8}$$

Here ρ_m,ρ_λ and p_λ are the functions of cosmic time t only.

Now with the help of Eqn. (8), the field equations (2) and (3) for the metric in Eqn. (1) can be written as

$$2\frac{\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}+\frac{1}{B^2}+\frac{\omega\dot{\phi}^2}{2\phi^2}+\frac{\ddot{\phi}}{\phi}+2\frac{\dot{\phi}\dot{B}}{\phi B}=-\frac{8\pi p_\lambda}{\phi} \tag{9}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}+\frac{\omega\dot{\phi}^2}{2\phi^2}+\frac{\ddot{\phi}}{\phi}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)=-\frac{8\pi p_\lambda}{\phi} \tag{10}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{1}{B^2}-\frac{\omega\dot{\phi}^2}{2\phi^2}+\frac{\dot{\phi}}{\phi}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)=\frac{8\pi(\rho_m+\rho_\lambda)}{\phi} \tag{11}$$

$$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)=\frac{8\pi}{\phi(3+2\omega)}(\rho_\lambda-3p_\lambda+\rho_m)\,. \tag{12}$$

Also, the energy conservation equation leads to

$$\dot{\rho}_m+\dot{\rho}_\lambda+3\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)(\rho_m+\rho_\lambda+p_\lambda)=0 \tag{13}$$

Here, the overhead dot denotes differentiation with respect to t.

3. Solutions of Field Equations

The field equations (9)-(12) are a system of four independent equations with six unknowns $A, B, p_\lambda, \rho_\lambda, \rho_m$ and ϕ. In order to get a

deterministic solution, we take the following two plausible physical conditions:
(i) The shear scalar σ is proportional to scalar expansion θ, which leads to the following relationship between the metric potentials,

$$A = B^m \tag{14}$$

Where, m, for $m \neq 1$, is a constant.
(ii) Scalar field ϕ is a function of average scale factor 'a' [34], i.e.,

$$\phi = \phi_0 a^n \tag{15}$$

Where ϕ_0 and n are arbitrary constants.

From Eqns. (9), (10) and (14), we get

$$\frac{\ddot{B}}{B} + (m+1)\frac{\dot{B}^2}{B^2} - \frac{1}{B^2(m-1)} + \frac{\dot{B}}{B}\frac{\dot{\phi}}{\phi} = 0. \tag{16}$$

From Eqns. (15) and (16), we get

$$\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2}\left(\frac{3(m+1)+n(m+2)}{3}\right) - \frac{1}{(m-1)B^2} = 0, \quad m \neq \pm 1. \tag{17}$$

From Eqns. (14) and (17), we get

$$A = (c_1 t + c_2)^m$$
$$B = c_1 t + c_2 \tag{18}$$

Where $c_1 = \left[\frac{3}{(m-1)\left[3(m+1)+n(m+2)\right]}\right]^{1/2}$ and c_2 is an integrating constant.

Thus, the metric (1) can be written as

$$ds^2 = dt^2 - (c_1 t + c_2)^{2m} dr^2 - (c_1 t + c_2)^2 (d\theta^2 + \sin^2\theta d\phi^2) \tag{19}$$

and the scalar field ϕ is given by

$$\phi = \phi_0 (c_1 t + c_2)^{\frac{n(m+2)}{3}}. \tag{20}$$

Here we are considering the minimally interacting matter and holographic dark energy components. Hence both the components conserve separately, so that we have [35,36]

$$\dot{\rho}_m + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\rho_m = 0 \tag{21}$$

$$\dot{\rho}_\lambda + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)(\rho_\lambda + p_\lambda) = 0. \tag{22}$$

From equations (9)-(11), (18) and (22), we get

$$\rho_m = \frac{c_3}{(c_1 t + c_2)^{m+2}} \tag{23}$$

$$\rho_\lambda = \frac{\phi_0 (c_1 t + c_2)^{\frac{n(m+2)-6}{3}}}{144\pi}\left\{c_1\left(18(1+2m) - n(m+2)^2(\omega n - 6)\right) + 18\right\} - \frac{c_3}{(c_1 t + c_2)^{m+2}} \tag{24}$$

$$p_\lambda = \frac{-\phi_0 (c_1 t + c_2)^{\frac{n(m+2)-6}{3}}}{144\pi}\left\{c_1\left(n^2(m+2)^2(\omega+2) + n(m+2)(m+1) + 9(1+m^2)\right) + 9\right\} \tag{25}$$

Fig. 1 describes the behavior of energy density (ρ_m) and holographic dark energy density (ρ_λ) versus time. It is understood that the energy density of ordinary matter and holographic dark energy are positive decreasing functions of time t and vanish for sufficiently large values of time.

Fig. 2 depicts the variation of equation of state (EoS) parameter (ω_λ) versus cosmic time (t). We observed that in early stage of evolution of the Universe, the EoS parameter ω_λ is positive (i.e., the Universe represents matter dominated phase) and at late time it is evolving with negative value (i.e., at the present time). The earlier real matter later on converted to the dark energy dominated phase of the Universe.

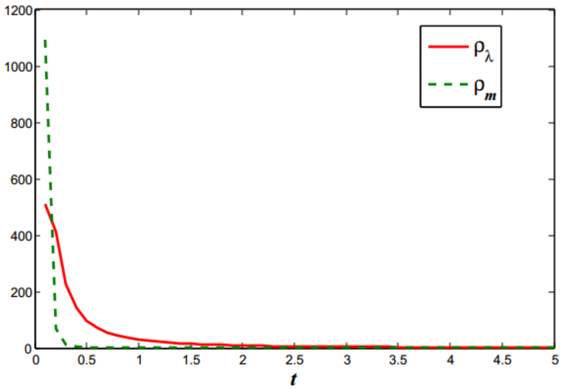

Fig.1: The plot of energy densities for holographic dark energy (ρ_λ) and ordinary matter (ρ_m) versus t for m=2, n=0.2 and ω=2.

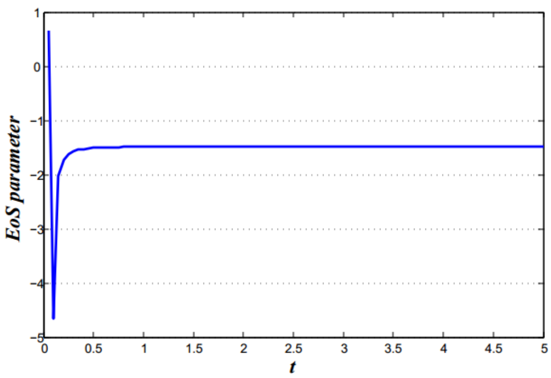

Fig.2: The plot of EoS parameter (ω_λ) versus t for m=2, n=0.2 and ω=2.

4. Some Other Important Properties of the Models

The spatial volume and average scale factor of the model in Eqn. (20) are given by

$$V=\sqrt{-g}=(c_1t+c_2)^{m+2}\sin\theta \qquad (26)$$

$$a=\left\{(c_1t+c_2)^{m+2}\sin\theta\right\}^{\frac{1}{3}} \qquad (27)$$

The expression for the expansion scalar θ is given by

$$\theta=\frac{c_1(m+2)}{(c_1t+c_2)} \qquad (28)$$

The Hubble parameter is given by

$$H=\frac{c_1(m+2)}{3(c_1t+c_2)} \qquad (29)$$

The shear scalar is given by

$$\sigma^2=\frac{7}{18}\left(\frac{c_1(m+2)}{c_1t+c_2}\right)^2 \qquad (30)$$

The deceleration parameter is given by

$$q=-1+\frac{3}{m+2} \qquad (31)$$

A positive sign of the deceleration parameter q indicates the standard decelerating model, whereas the negative sign indicates the inflating model. Recent observations show that the deceleration parameter of the model is in the range of $-1 < q < 0$ and the present day universe is undergoing an accelerated expansion. From Eqn. (31), it is observed that deceleration parameter q is in the range (-1,0) for $m>1$ and hence the model represents accelerated expansion of the universe.

The mean anisotropic parameter is given by

$$A_m=\frac{8(m-1)^2}{3(m+2)^2} \qquad (32)$$

The Jerk parameter is given by

$$J=\frac{(m-1)(m-4)}{(m+2)^2} \qquad (33)$$

The Look-back time is given by

$$\Delta t=H_0^{-1}\frac{(m+2)}{3}\left[1-(1+z)^{\frac{-3}{m+2}}\right] \qquad (34)$$

as $z\to\infty$, we get present age of the Universe i.e., $t_0=\frac{H_0^{-1}(m+2)}{3}$.

The Luminosity distance is given by

$$D=\frac{(m+2)}{m-1}H_0^{-1}\left(1-(1+z)^{\frac{m-1}{m+2}}\right)(1+z) \qquad (35)$$

5. Conclusions

Here, we have presented spatially homogeneous and anisotropic Kantowski-Sachs holographic dark energy model in Brans-Dicke theory of gravitation.

- The volume of the model is vanishing at $t = t_*$ where $t_* = -\frac{c_2}{c_1}$ and expansion scalar θ is infinite, which shows that the Universe starts evolving with zero volume at $t = t_*$ with an infinite rate of expansion. The pressure, energy density and shear scalar diverge at $t = t_*$.
- As $t \to \infty$, the scale factor and volume become infinite whereas σ, θ tend to zero. Thus the rate of expansion slows down with the increase of time.
- For our model the EoS parameter ω_λ is positive i.e., the Universe was matter dominated in early stage but in late time, the Universe is evolving with negative values i.e., the present epoch (see Fig. 2). Also the EoS parameter is in good agreement with the limit of latest observational results [37,38]. Thus our holographic dark energy model represents realistic model.
- Our model represents accelerating universe and deceleration parameter q is in good agreement with the recent observational data for $m>1$. For $m \neq 1$, the model is anisotropic throughout the evolution of the Universe. Also, we have obtained the expressions for look back time and luminosity distance versus red-shift.

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