Bianchi Tilted Cosmological Models in Lyra Manifold

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In this paper, we have obtained and presented tilted Bianchi cosmological models in the frame work of a scalar-tensor theory proposed by Sen (1957) based on Lyra (1951) geometry. To obtain a solution, it is assumed that the expansion scalar is proportional to the shear scalar, that is, $\theta \propto \sigma^2$, which leads to a relation between metric potentials. The physical and geometrical aspects of the results obtained are also discussed.

1. Introduction

In recent years, there has been a considerable interest in investigating spatially homogeneous and anisotropic universes in which the matter does not move orthogonally to the hyper surface of homogeneity. These are called tilted universes. Such tilted cosmological models help to study the effect of large scale peculiar velocity field relative to cosmic microwave background radiation. Further tilted cosmological models help to predict the growth of inhomogeneous models and also it establishes the relationship with the observed large scale structure. The general dynamics of tilted universes have been studied in detail by King and Ellis [3], Ellis and King [4], Collins and Ellis [5]. Bali and Sharma [6] have discussed tilted Bianchi type-*I* dust fluid cosmological model in general relativity. Pradhan and Rai [7] revisited tilted Bianchi type-*I* cosmological models filled with disordered radiation in general relativity investigated by Bali and Meena [8]. Sahu [9] has investigated tilted Bianchi type-VI_0 cosmological model in Saez-Ballester scalar tensor theory of gravitation. Sahu [10] has studied tilted LRS Bianchi type-*I* mesonic stiff fluid cosmological model. Anita [11] has obtained tilted Bianchi type-*IX* dust fluid cosmological model in general relativity. Bali and Kumawat [12] have discussed LRS Bianchi type-*II* tilted barotropic fluid cosmological model with heat conduction in general relativity. Sahu et al. [13] have studied tilted Bianchi type-*III* cosmological model in Lyra geometry. Sahu et al. [14] have discussed tilted Bianchi type-*III* wet dark fluid cosmological model in Saez-Ballester theory of gravitation. Recently, Sahu et al. [15] have obtained tilted Bianchi type-VI_0 wet dark fluid cosmological model in general relativity.

From last few decades there has been a lot of interest in scalar tensor theories of gravitation. Noteworthy among them is Sen's theory, a scalar-tensor theory proposed by Sen [1] based on Lyra [2] geometry. Halford [16] has shown that the scalar-tensor treatment based on Lyra's geometry predicts the same effects as in general relativity. Several authors have studied cosmological models within the framework of Lyra geometry with a constant gauge vector in the time direction. Recently, a lot of work has been done on string cosmological models and thick domain walls in Lyra geometry. Rao and Vinutha [17] have studied axially symmetric cosmological models in a scalar tensor theory of gravitation based on Lyra geometry. Bali et al. [18] have studied Bianchi type-*IX* barotropic fluid model with time-dependent displacement vector in Lyra geometry. Samanta and Debata [19] have studied five dimensional Bianchi type-*I* string cosmological models in Lyra manifold. Singh and Sharma [20] have discussed anisotropic dark energy Bianchi type-*II* cosmological models in Lyra geometry. Recently, Rao and Prasanthi [21] have obtained Kantowski-Sachs bulk viscous string cosmological models in Lyra geometry.

Bianchi type cosmological models are important in the sense that these are homogeneous and anisotropic, from which the process of isotropization of the Universe is studied through the passage of time and these space - times play a vital role in understanding and description of the early stages of evolution of the Universe. The simplicity of the field equations and relative ease of solutions made Bianchi space-times useful in constructing models of spatially homogeneous and

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anisotropic cosmologies (Ellis and Maccallum [22], Ryan and Shepley [23]). Also the anomalies found in the cosmic microwave background (CMB) and large scale structure observations stimulated a growing interest in anisotropic cosmological models of the universe.

The study of familiar solutions like FRW universe with positive curvature, the de Sitter universe, the Taub-Nut solutions and others correspond to Bianchi type-*II*, *VIII* and *IX* space-times. Rao et al. [24-26] have studied Bianchi type-*II*, *VIII* and *IX* cosmological models in different theories of gravitation. Rao and Vijaya Santhi [27] have obtained Bianchi type-*II*, *VIII* and *IX* magnetized cosmological models in Brans-Dicke theory of gravitation. Rao and Sireesha [28,29] have studied Bianchi type-*II*, *VIII* and *IX* string cosmological models with bulk viscosity in some scalar tensor theories of gravitation. Recently, Rao et al. [30] have investigated Bianchi type-*II*, -*VIII* and -*IX* perfect fluid cosmological models in modified theory of gravitation.

Motivated by above investigations and discussions about tilted cosmological models and Bianchi type-*II*, *VIII* and *IX* metrics, we have obtained and presented tilted Bianchi type-*II*,*VIII* and *IX* cosmological models in Sen theory based on Lyra geometry. The plan of the paper as follows: In Sec. 2, we discuss metric, energy momentum tensor and field equations. In Sec. 3, we have obtained solutions of the field equations. In Sec. 4, we discuss some important properties of models obtained. The last section contains some conclusions.

2. Metric and Energy Momentum Tensor

We consider spatially homogeneous Bianchi type-*II*, -*VIII* and- *IX* metrics in the form

$$ds^2 = -dt^2 + R^2\left[d\theta^2 + f^2(\theta)d\varphi^2\right] + S^2\left[d\psi + h(\theta)d\varphi\right]^2 \tag{1}$$

Where θ, φ and ψ are Eulerian angles. Also, R and S are functions of t only.

It represents

Bianchi type-*II* if $f(\theta)=1$ and $h(\theta)=\theta$

Bianchi type-*VIII* if $f(\theta)=Cosh\,\theta$ and $h(\theta)=Sinh\,\theta$

Bianchi type-*IX* if $f(\theta)=Sin\,\theta$ and $h(\theta)=Cos\theta$

The field equations given by Sen [1] are

$$R_{ij} - \frac{1}{2}Rg_{ij} + \frac{3}{2}\phi_i\phi^i - \frac{3}{4}g_{ij}\phi_s\phi^s = -8\pi T_{ij} \tag{2}$$

Where, T_{ij} is the energy momentum tensor, ϕ_i is the displacement field and the other symbols have their usual meaning as in Riemannian geometry. The displacement field ϕ_i can be written as $\phi_i = (0,0,0,\beta(t))$.

The energy momentum tensor for a perfect fluid distribution with heat conduction is given by

$$T^i_j = (\rho + p)u_i u^j + pg^j_i + q_i u^j + u_i q^j \tag{3}$$

and

$$q_i q^i > 0\ ,\ q_i u^i = 0 \tag{4}$$

Where, p is the pressure and ρ is the energy density of the perfect fluid distribution, q_i is the heat conduction vector orthogonal to u_i. The fluid vector u_i has the components $u^i = \left(\frac{\sinh\lambda}{R}, 0, 0, \cosh\lambda\right)$ satisfying Eqn. (4) and λ is the tilt angle. Here comma and semicolon denote ordinary and co-variant differentiation, respectively.

Using Eqns. (3) and (4), the field equations (2) for the metric in Eqn. (1) can be written as

$$\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} + \frac{S^2}{4R^4} + \frac{3}{4}\beta^2 = -8\pi\left\{(\rho + p)\sinh^2\lambda + p + 2q_1\left(\frac{\sinh\lambda}{R}\right)\right\} \tag{5}$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2 + \delta}{R^2} - \frac{3S^2}{4R^4} + \frac{3}{4}\beta^2 = -8\pi p \tag{6}$$

$$2\frac{\dot{R}\dot{S}}{RS}+\frac{\dot{R}^2+\delta}{R^2}-\frac{S^2}{4R^4}-\frac{3}{4}\beta^2=-8\pi\left\{-(\rho+p)\cosh^2\lambda+p-2q_1\left(\frac{\sinh\lambda}{A}\right)\right\} \quad (7)$$

$$(\rho+p)A\sinh\lambda\cosh\lambda+q_1\cosh\lambda+q_1\left(\frac{\sinh^2\lambda}{\cosh\lambda}\right)=0 \quad (8)$$

Here the over head dot denotes differentiation with respect to t .

3. Solutions of the Field equations

The field equations (5) to (8) are only four independent equations with seven unknowns R , S , ρ , p , λ , q_1 and β. So, in order to get a deterministic solution we take the following plausible physical conditions:

i. The shear scalar σ is proportional to scalar expansion θ, which leads to the linear relationship between the metric potentials R and S , i.e.,

$$R=S^k, \quad \text{where } k\neq 1 \quad (9)$$

ii. Stiff fluid condition i.e.,

$$p=\rho \quad (10)$$

iii. $$\beta=\beta_0 t^n \quad [31] \quad (11)$$

Where, *n* and *k* are arbitrary constants.

From Eqns. (5), (7), (9) and (10), we get

$$\frac{\ddot{S}}{S}+2k\frac{\dot{S}^2}{S^2}+\frac{\delta}{(k+1)}S^{-2k}=0 \quad (12)$$

Bianchi type-II ($\delta=0$) cosmological model

If, $\delta=0$, from Eqn. (12), we get

$$\frac{\ddot{S}}{S}+2k\frac{\dot{S}^2}{S^2}=0 \quad (13)$$

From Eqn. (13), we get

$$S=\left(k_3t+k_4\right)^{\frac{k_1}{k_3}} \quad (14)$$

Where, $k_3=\left(2k+1\right)k_1,\ k_4=\left(2k+1\right)k_2$ and $k_1\neq 0$, k_2 are integrating constants.

From Eqns. (9) and (14), we get

$$R=\left(k_3t+k_4\right)^{\frac{kk_1}{k_3}} \quad (15)$$

Now, the metric in Eqn. (1) can be written as

$$ds^2=-dt^2+\left(k_3t+k_4\right)^{\frac{kk_1}{k_3}}\left(d\theta^2+d\varphi^2\right)+\left(k_3t+k_4\right)^{\frac{k_1}{k_3}}\left(d\psi+\theta\, d\varphi\right)^2 \quad (16)$$

From Eqns. (6), (11), (14) and (15), we get

$$\rho=p=\frac{1}{8\pi}\left[\frac{3}{4}\left(k_3t+k_4\right)^{\frac{2k_1}{k_3}(1-2k)}+\frac{kk_1(2k_3-3kk_1)}{\left(k_3t+k_4\right)^2}-\frac{3}{4}\beta_0{}^2t^{2n}\right] \quad (17)$$

Thus the metric (Eqn. (16)) together with Eqn. (17) constitutes a spatially homogeneous and anisotropic Bianchi type-*II* tilted cosmological model in Sen's theory based on Lyra geometry.

From Eqns. (7), (8) and (17), we get

$$q_1=\frac{1}{8\pi}\left[\frac{kk_1(3kk_1-2k_3)}{\left(k_3t+k_4\right)^2}-\frac{3}{4}\left(k_3t+k_4\right)^{\frac{2k_1}{k_3}(1-2k)}+\frac{3}{4}\beta_0{}^2t^{2n}\right]\left(k_3t+k_4\right)^{\frac{kk_1}{k_3}}tanh2\lambda\cos h\lambda \quad (18)$$

$$\lambda = \frac{1}{2}\cosh^{-1}\left\{ \frac{\frac{kk_1(3kk_1 - 2k_3)}{(k_3t+k_4)^2} - \frac{3}{4}(k_3t+k_4)^{\frac{2k_1}{k_3}(1-2k)} + \frac{3}{4}\beta_0{}^2t^{2n}}{\frac{k_1{}^2(k^2+k+1) - k_1k_3(k+1)}{(k_3t+k_4)^2} + \frac{(k_3t+k_4)^{\frac{2k_1}{k_3}(1-2k)}}{4} + \frac{3}{4}\beta_0{}^2t^{2n}} \right\} \quad (19)$$

From Fig. 1, it is observed that the tilt angle doesn't vanish at any time and reaches a constant value for large values of time. So, the model obtained is tilted throughout the evolution of the Universe.

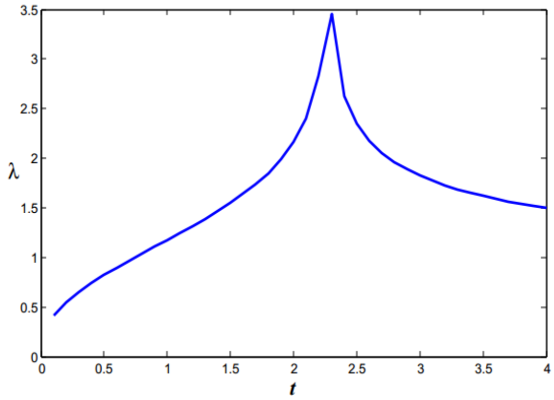

Fig.1: Plot of tilt angle versus time *t*

Fig. 2 demonstrates the behavior of pressure and heat conduction vector versus time. It is observed that both *p* and q_1 are decreasing functions of time and vanishes for large values of time.

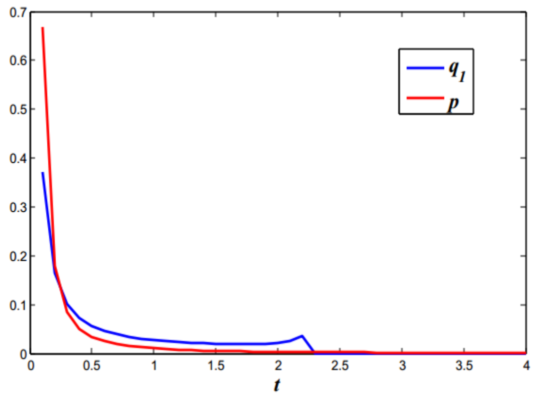

Fig.2: Plot of heat conduction vector and pressure versus t

Bianchi type-VIII ($\delta = -1$) cosmological model

If $\delta = -1$, from Eqn. (12), we get

$$\frac{\ddot{S}}{S} + 2k\frac{\dot{S}^2}{S^2} - \frac{1}{(k+1)}S^{-2k} = 0 \quad (20)$$

From Eqn. (20), we get

$$S = (k_6t + k_7)^{1/k} \quad (21)$$

Where, $k_6 = \frac{k}{(k+1)}$, $k \neq -1, k_7 = kk_5$ and k_5 is an integration constant.

From Eqns. (9) and (21), we get

$$R = (k_6t + k_7) \quad (22)$$

Now the metric (1) can be written as

$$ds^2 = -dt^2 + (k_6t+k_7)^2(d\theta^2 + d\varphi^2) + (k_6t+k_7)^{2/k}(d\psi + \theta\, d\varphi)^2 \quad (23)$$

From Eqns. (6), (21) and (22), we get the energy density and the pressure as

$$\rho = p = \frac{1}{8\pi}\left[\frac{3}{4}(k_6t+k_7)^{\frac{2}{k}-4} + \frac{1-k_6{}^2}{(k_6t+k_7)^2} - \frac{3}{4}\beta_0{}^2t^{2n}\right] \quad (24)$$

Thus the metric (Eqn. (23)) together with Eqn. (24) constitutes a spatially homogeneous and anisotropic Bianchi type-*VIII* tilted cosmological model in Lyra manifold.

From Eqns. (7), (8) and (24), we obtained heat conduction vector and tilt angle as

$$q_1 = \frac{1}{8\pi}\left[\frac{3}{4}\beta_0{}^2 t^{2n} + \frac{k_6{}^2 - 1}{(k_6 t + k_7)^2} - \frac{3}{4}(k_6 t + k_7)^{\frac{2}{k}-4}\right](k_6 t + k_7)(\tanh 2\lambda)(\cosh\lambda) \tag{25}$$

$$\lambda = \frac{1}{2}\cosh^{-1}\left\{\frac{\frac{3}{4}\beta_0{}^2 t^{2n} - \frac{3}{4}(k_6 t + k_7)^{\frac{2}{k}-4} + \frac{k_6{}^2 - 1}{(k_6 t + k_7)^2}}{\frac{3}{4}\beta_0{}^2 t^{2n} + \frac{1}{4}(k_6 t + k_7)^{\frac{2}{k}-4} + \frac{k_6{}^2}{k^2 (k_6 t + k_7)^2}}\right\} \tag{26}$$

From Fig. 3, we observed that the tilt angle is increased and then decreased rapidly at initial epoch and finally tends to zero for large values of time. Fig. 4 explains the behavior of pressure versus time. It is observed that pressure is decreasing function of time and vanishes for large values of time *t*. From Fig. 5, it is observed that heat conduction vector initially decreases rapidly and vanishes for large values of time *t*.

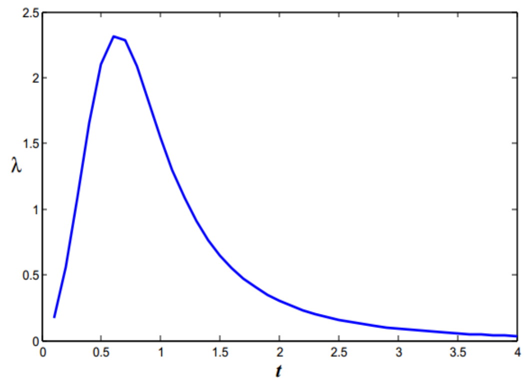

Fig.3: Plot of tilt angle versus *t*

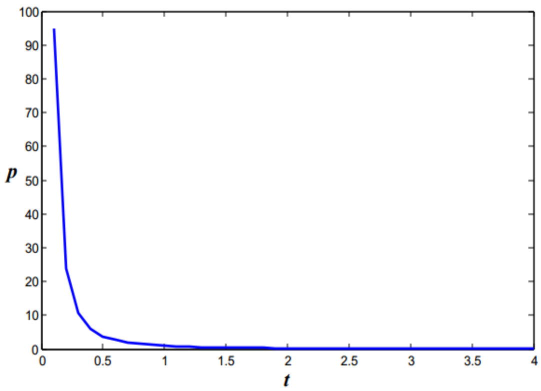

Fig.4: Plot of pressure versus *t*

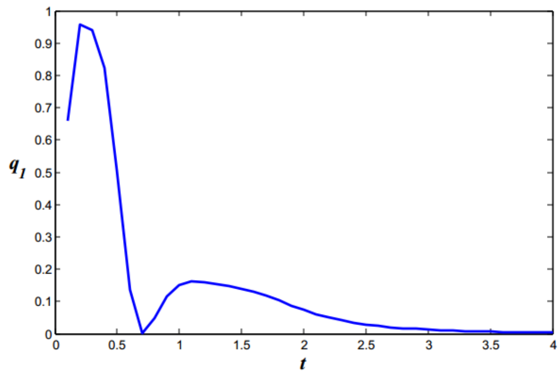

Fig.5: Plot of heat conduction vector versus *t*

Bianchi type-IX ($\delta = 1$) cosmological model

Here, it is observed that the model in this case (i.e., for $\delta = 1$**)** is similar to the model obtained in the case of Bianchi type-*VIII* , where $k_6{}^2 = \frac{-1}{(k+1)^2}$.

But, in this case we get imaginary solution of the model.

4. Some Other Important Features of the Models

Bianchi type-*II* cosmological model ($\delta = 0$)

The spatial volume for the model (16) is

$$V = T^{\frac{k_1}{k_3}(2k+1)} \tag{27}$$

Where, $T = (k_3 t + k_4)$.

The expression for expansion scalar θ is given by

$$\theta = \frac{k_1(2k+1)}{T}\left(\frac{2c_4 T^{-2} - T^{c_2} + 3\beta_0{}^2 t^{2n}}{2c_3 T^{-2} + T^{c_2} + 3\beta_0{}^2 t^{2n}}\right)^{\frac{1}{2}} + (\cosh\lambda)' \tag{28}$$

The deceleration parameter q is given by

$$q=\frac{d}{dt}\left(\frac{1}{H}\right)-1=2 \tag{29}$$

The non-vanishing shear scalars are given by

$$\sigma_{11}=T^{\left(\frac{2kk_1}{k_3}-1\right)}\left\{\frac{kk_1}{k_3}-\left(\frac{k_1(2k+1)}{3}(\cosh\lambda)+(\cosh\lambda)'\right)\left(\frac{2(c_1-c_3)T^{-2}-6T^{c_2}-3\beta_0{}^2t^{2n}}{2c_3T^{-2}+4T^{c_2}+3\beta_0{}^2t^{2n}}\right)\right\} \tag{30}$$

$$\sigma_{22}=\left(k_1\left(\frac{2k+4}{3}\right)(\cosh\lambda)+(\cosh\lambda)'\right)T^{\left(\frac{2k_1}{k_3}-1\right)} \tag{31}$$

$$\sigma_{44}=\left(\frac{k_1(2k+1)}{T}\cosh\lambda+(\cosh\lambda)'\right)\left(\frac{2(c_4+c_3)T^{-2}+3T^{c_2}+6\beta_0{}^2t^{2n}}{6\left(2c_3T^{-2}+4T^{c_2}+3\beta_0{}^2t^{2n}\right)}\right)+(\cosh\lambda)' \tag{32}$$

$$\sigma_{14}=\frac{1}{2}T^{\frac{kk_1}{k_3}-1}\left(T(\sinh\lambda)'-kk_1\sinh\lambda\right)+\frac{1}{6}\left(\frac{k_1(2k+1)}{T}\cosh\lambda+(\cosh\lambda)'\right)T^{\frac{kk_1}{k_3}}\sinh 2\lambda \tag{33}$$

The rotational vector is given by

$$\omega_{14}=T^{\frac{kk_1}{k_3}-1}\left(kk_1\sinh\lambda+T(\sinh\lambda)'\right) \tag{34}$$

Where

$$\sinh\lambda=\left(\frac{2\left(c_1T^{-2}-T^{c_2}\right)}{2c_3T^{-2}+T^{c_2}+3\beta_0{}^2t^{2n}}\right)^{\frac{1}{2}},\ \cosh\lambda=\left(\frac{2c_4T^{-2}-T^{c_2}+3\beta_0{}^2t^{2n}}{2c_3T^{-2}+T^{c_2}+3\beta_0{}^2t^{2n}}\right)^{\frac{1}{2}}$$

$$(\cosh\lambda)'=\frac{\left\{\frac{15}{4}\beta_0{}^2t^{2n-1}\left[2nT-k_3c_2t\right]T^{c_2-1}+12(c_3-c_4)(nT+k_3t)T^{-3}-\frac{k_3}{2}(c_2+2)(4c_4+c_3)T^{c_2-3}\right\}}{\left(c_4T^{-2}-\frac{1}{2}T^{c_2}+\frac{3}{2}\beta_0{}^2t^{2n}\right)^{\frac{1}{2}}\left(c_3T^{-2}+2T^{c_2}+\frac{3}{2}\beta_0{}^2t^{2n}\right)^{\frac{3}{2}}}$$

$$(\sinh\lambda)'=\frac{\left\{2c_3k_3T^{-5}+\frac{3}{2}\beta_0{}^2t^{2n}\left[\frac{2n}{t}\left(c_2T-c_1T^{-2}\right)-c_2k_3T^{c_2-1}-2k_3T^{-3}\right]-T^{c_2-3}\left[c_2(c_1+c_3)+2c_3+4\right]\right\}}{\left(c_1T^{-2}-T^{c_2}\right)^{\frac{1}{2}}\left(c_3T^{-2}+2T^{c_2}+\frac{3}{2}\beta_0{}^2t^{2n}\right)^{\frac{3}{2}}}$$

$c_1=kk_1(2kk_1-k_3)-k_1^2(k+1)+k_1k_3$, $c_2=\frac{2k_1}{k_3}(1-2k)$, $c_3=2\left[k_1^2\left(k^2+1\right)-k_1k_3(k+1)+kk_1^2\right]$,

$c_4=kk_1(4kk_1-3k_3)+k_1^2(k+1)-k_1k_3$.

Bianchi type-VIII ($\delta = -1$) cosmological model

The spatial volume for the model (23) is

$$V = \tau^{2+\frac{1}{k}} \quad (35)$$

Where, $\tau = (k_6 t + k_7)$.

The expansion scalar θ is given by

$$\theta = \frac{k_6 (2k+1)}{k\tau}\left(\frac{2c_8\tau^{-2} - \tau^{c_6} + 3\beta_0{}^2 t^{2n}}{2c_7\tau^{-2} + \tau^{c_6} + 3\beta_0{}^2 t^{2n}}\right)^{\frac{1}{2}} + (\cosh\lambda)' \quad (36)$$

The deceleration parameter q for the model (Eqn. (23)) is given by

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = -1 + \frac{3k}{2k+1} \quad (37)$$

The non-vanishing shear scalars are given by

$$\sigma_{11} = \tau^{(2k-1)}\left\{k - \left(\frac{k_6(2k+1)}{3k}(\cosh\lambda) + (\cosh\lambda)'\right)\left(\frac{(c_5 - c_7)\tau^{-2} - 3\tau^{c_6} - \frac{3}{2}\beta_0{}^2 t^{2n}}{c_7\tau^{-2} + 2\tau^{c_6} + \frac{3}{2}\beta_0{}^2 t^{2n}}\right)\right\} \quad (38)$$

$$\sigma_{22} = \left(k_6\left(\frac{2k+4}{3k}\right)(\cosh\lambda) + (\cosh\lambda)'\right)\tau \quad (39)$$

$$\sigma_{44} = \left(\frac{k_6(2k+1)}{k\tau}(\cosh\lambda) + (\cosh\lambda)'\right)\left(\frac{2(c_8 + c_7)\tau^{-2} + 3\tau^{c_6} + 6\beta_0{}^2 t^{2n}}{3\left(2c_7\tau^{-2} + 4\tau^{c_6} + 3\beta_0{}^2 t^{2n}\right)}\right) + (\cosh\lambda)' \quad (40)$$

$$\sigma_{14} = \frac{1}{2}\tau^{k-1}\left(\tau(\sinh\lambda)' - k_6\sinh\lambda\right) + \frac{1}{6}\left(\frac{k_6(2k+1)}{k\tau}(\cosh\lambda) + (\cosh\lambda)'\right)\tau^k \sinh 2\lambda \quad (41)$$

The rotational tensor is given by

$$\omega_{14} = \tau^{k-1}\left(k_6\sinh\lambda + \tau(\sinh\lambda)'\right) \quad (42)$$

Where

$$\sinh\lambda = \left(\frac{2\left(c_5\tau^{-2} - \tau^{c_6}\right)}{2c_7\tau^{-2} + \tau^{c_6} + 3\beta_0{}^2 t^{2n}}\right)^{\frac{1}{2}}, \quad \cosh\lambda = \left(\frac{2c_8\tau^{-2} - \tau^{c_6} + 3\beta_0{}^2 t^{2n}}{2c_7\tau^{-2} + \tau^{c_6} + 3\beta_0{}^2 t^{2n}}\right)^{\frac{1}{2}}$$

$$(\cosh\lambda)' = \frac{\left\{\frac{15}{4}\beta_0{}^2 t^{2n-1}\left[2n\tau - \frac{k_6}{k}c_6 t\right]\tau^{c_6-1} + 12(c_7 - c_8)(n\tau + k_3 t)\tau^{-3} - \frac{k_6}{2k}(c_6+2)(4c_8 + c_7)\tau^{c_6-3}\right\}}{\left(c_8\tau^{-2} - \frac{1}{2}\tau^{c_2} + \frac{3}{2}\beta_0{}^2 t^{2n}\right)^{\frac{1}{2}}\left(c_7\tau^{-2} + 2\tau^{c_6} + \frac{3}{2}\beta_0{}^2 t^{2n}\right)^{\frac{3}{2}}}$$

$$(\sinh\lambda)' = \frac{\left\{\begin{array}{r}\dfrac{2c_7k_6\tau^{-5}}{k}+\dfrac{3}{2}\beta_0{}^2t^{2n}\left[\dfrac{2n}{t}\left(c_6\tau-c_5\tau^{-2}\right)-\left(\dfrac{c_6k_6\tau^{c_6-1}+2k_6\tau^{-3}}{k}\right)\right] \\ -\tau^{c_6-3}\left[c_6\left(c_5+c_7\right)+2c_7+4\right]\end{array}\right\}}{\left(c_5\tau^{-2}-\tau^{c_6}\right)^{\frac{1}{2}}\left(c_7\tau^{-2}+2\tau^{c_6}+\dfrac{3}{2}\beta_0{}^2t^{2n}\right)^{\frac{3}{2}}}$$

$$c_5 = k_6^2 - 1 - \frac{k_6^2}{k^2},\quad c_6 = \frac{2}{k} - 4,\quad c_7 = \frac{2k_6^2}{k^2},\ \text{and}\ c_8 = k_6^2 - 1 + \frac{k_6^2}{k^2}.$$

5. Conclusions

In this paper ,we have presented Bianchi types *II* and *VIII* tilt cosmological models in the frame work of a scalar-tensor theory proposed by Sen [1] based on Lyra [2] geometry. The following are the conclusions:

- The volume of Bianchi type-*II* model vanishes at $t = \frac{-k_4}{k_3}$ and increases as $t \to \infty$ and the model has no singularity for *k>0*. We observe that, the expansion scalar is decreasing function of time.
- The deceleration parameter *q* is positive for Bianchi type-*II* model and hence it represents decelerating universe. Viswakarma [32] has shown that decelerating models are also consistent with recent cosmic background observations made by WAMP. However, in spite of the fact that the Universe, in this case, decelerates in the standard way, it will accelerate in finite time due to cosmic re-collapse where the Universe in turns inflates "decelerates and then accelerates" (Nojiri and Odintsov [33]).
- The volume of Bianchi type-*VIII* model vanishes at $t = \frac{-k_7}{k_6}$ and increases as $t \to \infty$ and also the model is free from singularities for *k>0*.
- For Bianchi type-*VIII* model, the deceleration parameter *q<0* for k<1. This shows that the model represents accelerated expansion of the universe.
- In both these models, we find that the pressure, energy density, heat conduction vector of the fluid distribution and tilt angles are decreasing functions of time and vanish for large values of time.
- If $\beta_0 = 0$, then these Bianchi type-*II* and *VIII* models reduce to Bianchi type- *II* and *VIII* tilt cosmological models in general relativity.

Also, the Bianchi type-*II* tilt cosmological model in general relativity is entirely different from the model obtained by Bali and Kumawat [12]. Thus, the models presented here are expanding, shearing, rotating and anisotropic throughout the evolution of the Universe.

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