

Analysis of Rainfall Data of Ikeja, Southwestern Nigeria, Using Wavelet Transform

A. O. Adewale¹, A. A. Alabi^{2,*} and T. O. Lawal³

¹*Department of Science Laboratory Technology, Moshood Abiola Polytechnic, Abeokuta, Nigeria*

²*Department of Physics, Federal University of Agriculture, Abeokuta, Nigeria*

³*Department of Physics, University of Ilorin, Ilorin, Nigeria*

Continuous wavelet transform was applied on the total monthly rainfall data of Ikeja in an attempt to gain insight into the rainfall variability observed in the area. Apart from the analysis of the rainfall variability, study of the power concentration of the time series was carried out by obtaining the wavelet power spectrum of the time series. Study of the frequency components in the time series was also done using the global wavelet spectrum. The wavelet power spectrum showed that the power is more concentrated between 8-16-months band, an indication that the time series of study is characterised by strong annual signal. The global wavelet spectrum revealed that the monthly rainfall data of Ikeja comprised mainly an annual frequency. The 8–16-month band modulation was therefore investigated by an average of all scales, between 8 and 16 months, which is a measure of variation of average monthly variance with time where periods of low and high variance were identified

1. Introduction

Rainfall being one of major factors of climate influences agricultural activities, commercial activities, industrial activities and government national planning amongst several other government and private decisions that are capable of having direct impacts on citizenries of any nation. Ikeja, the capital of Lagos state, southwestern Nigeria houses Muritala Mohammed international airport which is one of Nigeria's major airports. The flood which has persistently ravaged Lagos state over the years is mainly influenced by more than abundant rainfall.

Since its theory was placed on a firm footing in 1984 by Grossman and Morlet [1], wavelet transform (WT) which is a relatively recent development in signal processing has provoked much interest in the research world. Though Fourier transform (FT) cannot be discarded from arsenal of tools of signal analysis, wavelet transform has been used as an alternative to FT in preserving local, non-stationary, multi-scaled features. One of its advantages is its ability to analyze temporal variability of different scales without requiring a stationary series. It is suitable for analyzing irregular distributed events and time series with non-stationary energy at many different scales or frequencies. WT, being a method of time-

scale analysis of a signal, is better than FT. This is because it offers high-resolution features as well as having good localization in time and scale domains coupled with kits ability to analyze signals of multi-time scales [2,3].

Wavelet as a tool for analysis has recorded applications in astronomy, acoustics, data compression, nuclear engineering, sub-band coding, signal and image processing, magnetic resonance imaging, speech discrimination, optics, fractals, radar, human vision, pure mathematics and geophysics such as tropical convection. The wavelet has also been used to gain insight into atmospheric cold fronts, temperature variability, and dispersion of ocean waves, wave growth and breaking, structure turbulent flows [4-8]. The following section describe the theory of wavelet transform, the rainfall data in question and application of wavelet to the data using Torrence and Compo's program [5]. The aim of this study is to gain insight into rainfall variability pattern of the study area, determine the power concentration of the time series (rainfall data) and study the frequency components in the time series using the global wavelet spectrum.

2. Data and Study Area

The monthly rainfall data of Ikeja, Lagos state south western Nigeria used in this study cover 50 years (1962-2012). The data was obtained from the archive of Nigeria Meteorological Agency. The

*derylab@yahoo.com

city of Ikeja, being capital of Lagos State lies in south-western Nigeria, on the Atlantic coast in the Gulf of Guinea, west of the Niger River delta, located on Latitude 6°35'39" N, Longitude 3°20'14" E and 41 m (134 ft.) above sea level. The Lagos state has a tropical wet and dry climate with two distinct rainy seasons; the more intense one occurs between April and July, with a milder one from October to November. At the peak of the rainy season the weather in Lagos is wet about half the time. Lagos experiences a dry season (when it rains less than two days per month) during August and September, as well as between December and March, accompanied by harmattan winds from the Sahara Desert, which are at their strongest from December to early February. The temperature range in Lagos is fairly small, generally staying between 91°F (33°C) and 70°F (21°C). The hottest month is March, when average daytime temperatures reach 84°F (29°C), while July is the coldest month with an average temperature of 77°F (25°C). The average temperature is 27°C with average annual rainfall of 1693mm. The lowest average temperature of 25.2°C of the region occurs in August. Variation between the driest and wettest month is 365mm. The least rainfall month is December with an average value of 21mm. The greatest amount of precipitation is recorded in June with average value of 386mm.

The climate of Lagos State like other parts of Nigeria is principally governed by the influence of three major wind currents; these are the maritime tropical (mT) air mass, the equatorial easterlies and the continental tropical (cT) air mass [5]. The maritime tropical air mass originates from the southern high-pressure belt located off the coast of Namibia, and in its trajectory, picks up moisture from over the Atlantic Ocean, crosses the Equator and enters south-western Nigeria. The cT air mass originates from the high-pressure belt North of the Tropic of Cancer. It picks up little moisture along its path and is thus dry [10]. The two air masses (mT and cT) intercept along a slanting boundary zone called the Inter-Tropical Discontinuity (ITD). The flow of the ITD is sun-synchronous [11], the equatorial easterlies are rather unpredictable cool

air masses, which come from the east and flow in the upper atmosphere along the ITD. The air mass dives down occasionally undercuts the mT or cT air mass and produces line squalls or dust devils [9].

3. Method

There exist numerous mathematical transformations which can be applied to gain insight into signals. One of such transforms is Fourier transform which is perhaps the most popular, however, FT has certain limitation which is its zero time localization. The wavelet transform developed to overcome this setback of FT can be applied in analyzing time series which are usually characterized by non-stationary power at many varying frequencies [5]. This is because the wavelet analysis simultaneously provides both time and frequency view of one dimensional time series. The consequence of this is that information on amplitude and time variation of such amplitude of any periodic signals concealed in the time series can be obtained. One of the mother wavelets (Morlet wavelet) used in signal analysis is shown in Fig. 1. These mother wavelets have the ability to be scaled (compressed or extended) to examine different sections of a signal.

The wavelet analysis always applies a wavelet of the same characteristic shape but different scales on a particular time series. Apart from information on the amplitude of any periodic signal that can be obtained by WT, it is also possible to obtain information on its phase if the mother wavelet used is a complex type. The wavelet displayed in Fig. 1 is a complex wavelet defined as

$$\Psi\Psi_0(\eta) = \pi^{-1/4}e^{i\omega_0\eta}e^{-\eta^2/2} \quad (1)$$

Where, $\Psi\Psi_0(\eta)$ is the wavelet value at non dimensional time η and ω_0 is the non-dimensional frequency whose value is 6 in this work in order to be an admissible wavelet, an admissible wavelet must have zero mean and have localization in time and frequency.

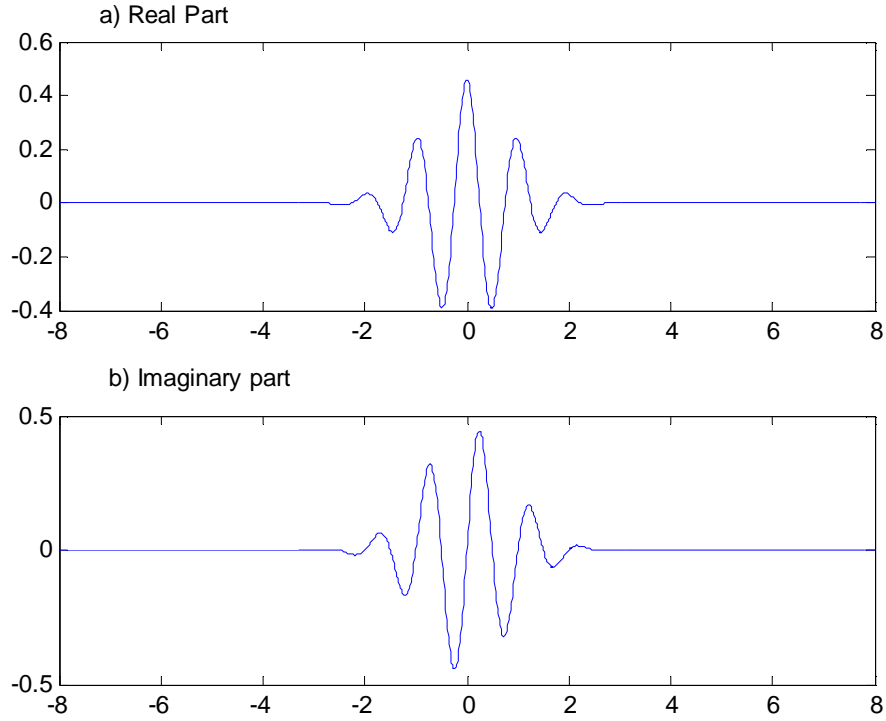


Fig.1: Morlet wavelet: (a) the real Part (b) the imaginary part

The scaled and shifted versions of the wavelet are defined by Eqn. (2) as:

$$\Psi\Psi\left[\frac{(n'-n)\delta t}{s}\right] = \left(\frac{\delta t}{s}\right)^{1/2} \Psi\Psi_0\left[\frac{(n'-n)\delta t}{s}\right] \quad (2)$$

Where, s is the scale or dilation parameter that measures how much the wavelet is stretched, n is the shift or translation parameter used to shift the wavelet in time. The factor $s^{-1/2}$ is introduced as normalization factor to ensure that the wavelet transforms at each scale has unit energy so as to ensure that they are directly comparable to each other and transforms of other time series. For a time series X , with values of x_n at time index n , with each value is separated in time by a constant time interval δt , the wavelet transform $W_n(s)$ is the convolution of the wavelet function with the original time series:

$$W_n(s) = \sum_{n=0}^{N-1} x_n \Psi^*\left[\frac{(n'-n)\delta t}{s}\right] \quad (3)$$

Where the asterisk represents complex conjugate. A new time series of the projection amplitude against time can be constructed by shifting this wavelet along the time series. By changing the width of the wavelet its scale can be varied. Relevant literatures (e.g., Torrence and Compo [5]) offer additional technicalities used in various mother wavelets.

4. Results and Discussions

4.1. Wavelet power spectrum

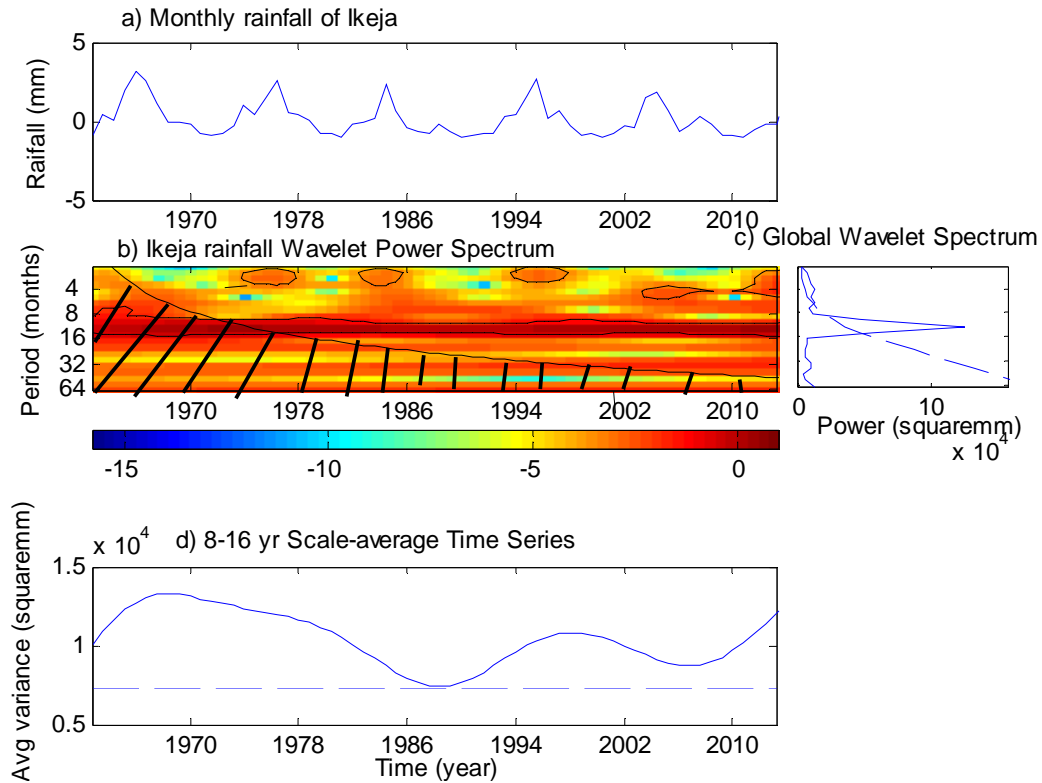


Fig.2: (a) Total monthly rainfall series of Ikeja. (b) The wavelet power spectrum. The contour levels are chosen so that 75%, 50%, 25%, and 5% of the wavelet power is above each level, respectively. Shaded region is the cone of influence, where zero padding has reduced the variance. Black contour is the 5% significance level, using a red noise ($\alpha = 0.72$) background spectrum. (c) The global wavelet power spectrum (blue line). The dashed line is the 5% significance level for the global wavelet spectrum. (d) Scale-average wavelet power over the 8–16-month band for the total monthly rainfall in Ikeja. The dashed line is the 95% confidence level assuming red noise $\alpha = 0.72$.

Monthly rainfall data are used in this study. Hence for an analysis using the wavelet, the parameters are $\delta t = 1\text{month}$ and $S_o = 2\delta t$, $\delta j = 0.25$ to do 4 sub-octaves per octave, $j_1 = 7/\delta j$ to do 7 powers-of-two with δj sub-octaves each. Fig. 2b represents the square of absolute values or power of the wavelet transform of the monthly rainfall covering 51 years in Ikeja which is shown in Fig. 2a. Information on the relative power of a particular scale and a particular time is provided by power of the monthly rainfall data. Apart from revealing the magnitudes of the scaled and shifted version of the mother wavelet, their actual oscillations are also revealed by Fig. 2b. Looking at the Fig. 2b, it is evident that power is more concentrated between the 8-16 month band, an indication that the time series of study is characterized by a strong annual signal. From the wavelet power spectrum of the Fig. 2b, it can be seen that there are more low frequencies peaks (for

example below 16 months). Wavelet power spectrum for rainfall with characteristic scale of 1-6 months show a significant peak above 5% level. 8-16 month band has variance of power which shows only wet years, when there is substantial power decrease in this band, it represents a dry year, a wet year is indicated when the power is maximum. For example wet periods can be identified throughout the entire years of the study, however a reduction in power not substantial can be observed between the year 1982 and the beginning of 1992. Another insignificant reduction in power is identified between the years 2002 and 2010. An increase in power can be identified in 2010 with the increase still continuing through 2012.

The region shaded with black lines in Fig. 2b is the cone of influence (COI). The cone of influence is the portion of the wavelet spectrum where effect of zero padding cannot be ignored as it has reduced the variance. Since the rainfall data in question is

finite-length time series, there will be error at the beginning and end of the wavelet power spectrum. A way of solving this problem is to pad the end of the data with zeroes before transforming using wavelet after which the zeroes are removed. Sufficient zeroes are used to pad the time series to take the total length N to the next higher power of 2. The consequence of this is that the edge effect is limited and the Fourier transform is faster. Discontinuities are introduced at the end parts as a result of padding with zeroes and hence the amplitude near the edges is decreased as larger scales are approached. The COI is the portion of the wavelet spectrum where edge effects become significant; it is defined as the e-folding time for the autocorrelation of wavelet power at each scale. The peak within this portion has been reduced in magnitude by the zero padding, thus it is not certain whether the decrease in a particular band power in this shaded region (COI) represents a true decrease in variance or an artifact of the padding. Wavelet power spectrum obtained using much narrower wavelets like Mexican hat wavelet produce much smaller COI and thus less influenced by edge effects. It is important to mention that it is not necessary padding cyclic series with zeroes, a consequence of which is absence of COI.

The contour with black edge existing between 1 and 6 months period in Fig. 2b is the 5% significance level, using a red noise background spectrum. Red-noise has the uni-variant lag-1 autoregressive process as its simple model. The time series when correlated with one unit time shifted version of itself is the lag-1. This would be a one month shift in this present study. The lag-1 is the estimate of persistence of an anomaly from a month to the next. Using an approximation $\alpha = (\alpha_1 + \alpha_2^{1/2}) / 2$, the true lag-1 α can be calculated. Here, α_1 represents the lag-1 autocorrelation while α_2 represents the lag-2 autocorrelation which equals lag-1 but having two points shift instead of one. In this present time series the true lag-1 of 0.72 is used. Having assumed that the time series has a mean power spectrum, the null hypothesis is defined for the power spectrum. Any peak in the wavelet power spectrum which is significantly above the background spectrum can be assumed to be a true feature having a certain percent confidence. 5% significance level is the same as the 95% confidence level which means a test against a certain background level. The interpretation of 95% confidence is that 5% of the wavelet power should be above this level.

4.2. Global wavelet power spectrum

The twelve month periodicity of this time series is corroborated by an integral of power over time (Fig. 2c). Only one important (significant) peak above the 95% confidence for the global wavelet spectrum is revealed (assuming $\alpha = 0.72$ represented by the dashed line) in Fig. 2c. Also, in Fig. 2c has a significant peak at the 5% level over 1-4 months band. Obviously, most peak monthly rainfall values for Ikeja coincide with pulses whose powers are significantly high within the 1-4 months band in Fig. 2b.

4.3. Scale-average time series

The scale average wavelet power shown in Fig. 2d represents the variation of the average variance with time within 8-16 months band. The figure (Fig. 2d) was obtained from the average over all scales within 8-16 month. It measures average year variance against time. This plot (Fig. 2d) clearly reveals periods when monthly rainfall variance was high e.g. from 1962 to 1986 and from 1992 to 2012 which distinctly indicate periods that are wetter than normal. Between 1986 and 1992, from the figure, were periods of normal wet season.

5. Conclusion

Wavelet analysis has been applied in an attempt to investigate the variability of the monthly rainfall time series of Ikeja. A strong power concentration between the 8-16 months band is shown by the wavelet power spectrum. This has revealed an amount of periodicity of rainfall events supported by the peak of global wavelet spectrum. The period of high monthly rainfall variance has been detected by the average of all scales within 8-16 months. This study can be furthered by way of using wavelet analysis to investigate water drainage flow with a few providing possible solution to flood problem in the study area. In this same manner, this method of analysis permits the analysis of other time series for a period of time of monthly years, such time series include wind speed, relative humidity, vapor pressure, sunshine hours etc.

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References

- [1] A. Grossman and J. Morlet, *SIAM J. Math. Anal.* **15**, 723 (1984).
- [2] B. D. Nener, T. A. Ridsdil-Smith and C. Zeisse, *Infrared Physics and Technology* **40**, 309 (1999).
- [3] T. Clemen, *Ecological Modelling*, **108**, 107 (1998).
- [4] A. Grape, *Computational Science and Engineering* **2(2)**, 50 (1995).
- [5] C. Torrence and G. P. Compo, *Bull. Am. Met. Soc.* **79(1)**, 61 (1998).
- [6] M. Farge, *Ann. Rev. Fluid Mech.* **24**, 395 (1992).
- [7] L. C. Smith, D. L. Turcotte and B. L. Isacks, *Hydrol. Processes* **12**, 233 (1998).
- [8] C. A. G. Santos, C. O. Galvao, K. Suzuki and R. M. Trigo, *Ann. J. Hydraul. Engang. JSCE*, **45**, 211 (2001).
- [9] N. P. Iloeje, *A New Geography of Nigeria*, new revised edition (Longman Publishers, London, 1981).
- [10] B. E. Omogbai, *J. Hum. Ecol.* **31(3)**, 185 (2010).
- [11] J. O. Ayoade, *J. Tropical Geography* **39**, 11 (1974).

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