Second harmonic generation in high density plasma

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In this paper, we have studied the mechanism of second harmonic generation by nonlinear effects. The propagation of a linearly polarized electromagnetic wave through homogeneous high density quantum plasma in the presence of transverse magnetic field is taken into consideration. The dispersion relations for the fundamental and the second harmonic frequencies have been derived using the quantum hydrodynamic (QHD) model. The effects of quantum Bohm potential and Fermi pressure have been taken into account. The second harmonic is found to be less dispersed than the first. This leads to generation of second harmonic radiation with significant conversion efficiency.

1. Introduction

The generation of harmonic radiation is significant in terms of laser-plasma interaction and has brought interesting notice due to the diversity of its applications. The odd harmonics of laser frequency are generated in the majority of laser interactions with homogenous plasma [1]. It has been remarked that second harmonic generation takes place in the presence of density gradient [2-3], which gives rise to perturbation in the electron density at the laser frequency. The second harmonic generation has been related with filamentation [4] and emitted in a direction perpendicular to the laser beam from filamentary structures in the under dense target corona.

It has been reported by Caruso et al [5] and Decroisette et al [6] regarding the generation of second harmonic wave in laser produced plasma. The origin was interpreted as the nonlinear effects in the interaction of laser radiation and the plasma produced therein. Experimental verification was also observed by some authors. Basov et al [7] gave a theoretical model for second harmonic generation. They calculated the mechanism of generation, intensity as well as other parameters like temperature, dimension of the produced plasma. They also cited from literature that the possibility of Langmuir waves in plasma arises at sufficiently high densities. Chandra et al [8] studied the propagation of surface waves in a relativistically degenerate plasma and reported the generation of second harmonic waves.

In dense plasmas, when the de Broglie wavelength of the charge carriers becomes comparable to the spatial scale of plasma system, the quantum effects start playing a crucial role on the dynamics of plasma particles and their study becomes important. The quantum plasma has received much attention in recent years due to its important applications in ultra-small electronics devices, quantum dots and quantum wire [9] in dense astrophysical plasma system [10-11] in laser-produced plasma [12], nonlinear optics [13] and X-ray FEL [14]. The dispersion with quantum effects corrections for linear waves in a uniform cold quantum plasma with non-zero external magnetic field have been studied by Ren et al. [15-16].

In the present paper, we present a study of second harmonic generation when a linearly polarized laser beam propagates through a homogeneous high density quantum plasma in the presence of a transverse magnetic field. The effect of quantum Bohm potential and Fermi pressure have taken in the account. The nonlinear current density and dispersion for the fundamental and second harmonic frequencies have been obtained in Sec. 2. Normalized wave amplitude of the second harmonic and its conversion efficiency are obtained in Sec 3. Discussion is presented in Sec. 4**.**

2. Formulation

Consider a linearly polarized laser beam propagating along the z direction in a uniform quantum plasma. The plasma is embedded in a constant magnetic field $\vec{b}(b_0 y)$. The electric vecotor of the laser field is given by

$$\vec{E}_l = \frac{1}{2}\hat{x}E_0 e^{i(k_o z-\omega_0 t)} + c.c. \quad (1)$$

where, ω_0 is the frequency and k_0 is the propagation constant of the laser. During the course of propagation of the laser beam through the magnetized plasma, a transverse current density at frequency $2\omega_0$ arises [17] and act as a source for second harmonic generation. The electric fields corresponding to the fundamental (ω_0) and the second harmonic $(2\omega_0)$ frequencies are assumed to be given as

$$\vec{E}_l = \frac{1}{2}\hat{x}E_0 e^{i(k_o z-\omega_0 t)} + c.c. \quad (2)$$

and

$$\vec{E}_l = \frac{1}{2}\hat{x}E_0 e^{i(k_o z-\omega_0 t)} + c.c. \quad (3)$$

respectively. The amplitudes E_1 and E_2 are considered to be z dependent. The propagation vector $\vec{k}_1 (= \omega_0\mu_1/c)$ and $\vec{k}_2(= \omega_0\mu_2/c)$ correspond to the frequencies ω_0 and $2\omega_0$ respectively, where μ_1 and μ_2 are the corresponding wave refractive indices.

The QHD equations governing the interaction between the electromagnetic field and plasma electron are [18-20]

$$\frac{d(\gamma\vec{v})}{dt} = -\frac{e\vec{E}}{m} - \frac{e}{mc}\left(\vec{v}\times\vec{B}\right) - \frac{\vec{v}_F^{\,2}}{3n_0^2}\frac{\nabla n^3}{n}$$

(4)

$$+\frac{\hbar^2}{2m}\nabla\left(\frac{1}{\sqrt{n}}\nabla^2\sqrt{n}\right) \quad (4)$$

and the continuity equation

$$\frac{\partial n}{\partial t} + \vec{\nabla}.(n\vec{v}) = 0 \quad (5)$$

Where, $\vec{v}$ is the velocity of plasma electron, n is the plasma electron density, $\vec{E} = \vec{E}_1 + \vec{E}_2$, γ is the relativistic factor, B is magnetic vector of the radiation field, $n(= n_0 + n^{(1)})$ is the electrons density, m is the electron's rest mass, $\hbar$ is the Planck's constant divided by 2π, $v_F\ (= (\hbar/m)(3\pi^2 n)^{1/3})$ is the Fermi velocity. The third term on the right–hand side of Eqn. (4) denotes the Fermi electron pressure. The fourth term is the quantum Bohm force and is due to the quantum corrections in the density fluctuation. The classical equation may be recovered in the limit of $\hbar = 0$. Initially, the plasma electrons are at rest as the plasma is cold and the external magnetic field does not contribute. By using perturbation expansion, we found from Eqn. (4) for the first order fields quantities and substituting the relevant quantities, gives the first order transverse and longitudinal velocities as

$$v_x^{(1)} = -\frac{i}{2}\frac{a_1 c\omega_0^2}{(\omega_0^2-\omega_c^2)}e^{i(k_1 z-\omega_0 t)} - \frac{i}{2}\frac{2ca_2}{(4\omega_0^2-\omega_c^2)}e^{i(k_2 z-\omega_0 t)}$$

$$+\frac{a_1(\omega_0+i\omega_c)\Omega_{q_1}}{(\omega_0^2-\omega_c^2)}n^{(1.1)} + \frac{a_2(2\omega_0+i\omega_c)}{(4\omega_0^2-\omega_c^2)}n^{(1,2)} + c.c.(6)$$

(6)

$$v_z^{(1)} = -\frac{a_1 c\omega_0\omega_c}{2(\omega_0^2-\omega_c^2)}e^{i(k_1 z-\omega_0 t)} - \frac{a_2 c\omega_0\omega_c}{2(4\omega_0^2-\omega_c^2)}e^{i(k_2 z-2\omega_0 t)}$$

$$+\frac{a_1(\omega_0-i\omega_c)\Omega_{q_1}}{(\omega_0^2-\omega_c^2)}n^{(1,1)} + \frac{a_2(2\omega_0-i\omega_c)}{(4\omega_0^2-\omega_c^2)}n^{(1,2)} + c.c.(7)$$

Where,

$$\Omega_{q_{(1,2)}} = \left\{\frac{v_F^2}{n_0} + \frac{\hbar^2}{4m^2}\frac{1}{n_0}k_{(1,2)}^2\right\}k_{(1,2)},$$

$a_1 = eE_1/mc\omega_0$, $a_2 = eE_2/mc\omega_0$ and $\omega_c = eb/mc$ is the cyclotron frequency of the plasma electron. While obtaining the above, it has been assumed that the first order perturbed electron density $n^{(1)}(= n^{(1,1)} + n^{(1,2)})$ vary as $n^{(1,1)} = \eta_1 e^{i(k_1 z-\omega_0 t)}$ and $n^{(1,2)} = \eta_2 e^{i(k_2 z-2\omega_0 t)}$ for fundamental and second harmonics respectively. Substituting, the perturbed electron trajectary into the continuity equation (Eq.(5)) gives the first order plasma electron density as

$$n^{(1)} = \frac{n_0 a_1 k_1}{(\omega_0^2-\omega_c^2)}\left[-\frac{c\omega_c}{2} + \frac{(\omega_0-i\omega_c)\Omega_{q_1}\eta_1}{\omega_0}\right]e^{i(k_1 z-\omega_0 t)}$$

$$+\frac{n_0 a_2 k_2}{\left(4\omega_0^2-\omega_c^2\right)}\left[-\frac{c\omega_c}{4}+\frac{(2\omega_0-i\omega_c)\Omega_{q_2}\eta_2}{2\omega_0}\right]e^{i(k_2 z-2\omega_0 t)}+c.c. \quad (8)$$

where (8WW

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$$\eta_1=\frac{-ck_1 n_0\omega_c\omega_0^2}{2(\omega_0^2\left(\omega_0^2-\omega_c^2\right)+n_0\omega_0 k_1(\omega_0-i\omega_c)\Omega_{q_1})}$$

and

$$\eta_2=\frac{-2ck_2 n_0\omega_c\omega_0^2}{2(4\omega_0^2\left(4\omega_0^2-\omega_c^2\right)+n_0\omega_0 k_2(2\omega_0-i\omega_c)\Omega_{q_2})}$$

.

Similarly, the second order transverse and longitudinal velocities are also obtained from Eqn. (4) as

$$v_x^{(2)}=\frac{a_1^2 k_1}{\left(4\omega_0^2-\omega_c^2\right)\left(\omega_0^2-\omega_c^2\right)^2}[-\frac{i}{4}c^2\omega_0^2\omega_c\left(\omega_0^2-4\omega_c^2\right)$$

$$+\left\{-\delta_1\Omega_{q_1}\eta_1+\delta_2\Omega_{q_2}\eta_1^2\right\}]e^{2i(k_1 z-\omega_0 t)}+c.c. \quad (9)$$

and

$$v_z^{(2)}=\frac{a_1^2 k_1}{\left(4\omega_0^2-\omega_c^2\right)\left(\omega_0^2-\omega_c^2\right)^2}[\ \frac{1}{4}c^2\omega_0\left\{3\omega_0^2\omega_c^2+\left(\omega_0^2-\omega_c^2\right)\left(4\omega_0^2-2\omega_c^2\right)\right\}$$

$$-\left\{\delta_3\Omega_{q_1}\eta_1+\delta_4\Omega_{q_1}^2\eta_1^2\right\}]e^{2i(k_1 z-\omega_0 t)}+c.c. \quad (10)$$

Where

$$\delta_1=\omega_0(\omega_0-i\omega_c)\left\{2i\left(\omega_0^2-\omega_c^2\right)+ic\left(\omega_0^2+\omega_c^2\right)+(\omega_0+i\omega_c)\left(\omega_0^2-\omega_c^2+c\omega_0\omega_c\right)\right\}$$

,

$$\delta_2=2\omega_0\left(\omega_0^2+\omega_c^2\right)+i\omega_c(\omega_0-i\omega_c)^2,$$

$$\delta_3=\omega_c(\omega_0-i\omega_c)\left\{\omega_0^2-\omega_c^2+\frac{c\omega_0^2}{2}\right\},$$

$$\delta_4=i\omega_0(\omega_0-i\omega_c)\left\{\omega_0^2-\omega_c^2+\frac{c\omega_c^2}{2}\right\}.$$

The velocities obtained above contain collective effects of the laser and the static magnetic fields on the plasma electrons. The second order perturbed electron density comes out be

$$n^{(2)}=\frac{n_0 k_1}{\omega_0}[\ \frac{c^2 a_1^2\omega_0 k_1\left\{3\omega_0^2\omega_c^2+\left(\omega_0^2-\omega_c^2\right)\left(4\omega_0^2-2\omega_c^2\right)\right\}}{4\left(\omega_0^2-\omega_c^2\right)^2\left(4\omega_0^2-\omega_c^2\right)}$$

$$-\frac{k_1}{\left(\omega_0^2-\omega_c^2\right)^2\left(4\omega_0^2-\omega_c^2\right)}\left\{\delta_3\Omega_{q_1}\eta_1+\delta_4\Omega_{q_1}^2\eta_1^2\right\}]e^{2i(k_1 z-\omega_0 t)}+c.c.$$

(11)

The transverse current density can now be obtained by substituting the plasma electron density and velocity

$$J_x=\left[\frac{i}{2}\frac{en_0 ca_1\omega_0^2}{\left(\omega_0^2-\omega_c^2\right)}-\frac{en_0 a_1(\omega_0+i\omega_c)}{\left(\omega_0^2-\omega_c^2\right)}\eta_1\Omega_{q_1}\right]e^{i(k_1 z-\omega_0 t)}$$

$$+\left[\frac{i}{2}\frac{2en_0 ca_2\omega_0^2}{\left(4\omega_0^2-\omega_c^2\right)}-\frac{en_0 a_2(2\omega_0+i\omega_c)}{\left(4\omega_0^2-\omega_c^2\right)}\eta_2\Omega_{q_2}\right]e^{i(k_1 z-\omega_0 t)}$$

$$+\frac{ek_1 n_0}{\left(4\omega_0^2-\omega_c^2\right)\left(\omega_0^2-\omega_c^2\right)^2}\left[-\frac{i}{4}a_1^2 c^2\omega_0^2\omega_c\left(\omega_0^2+\omega_c^2\right)\right.$$

$$-a_1^2\delta_1\eta_1\Omega_{q_1}+(\delta_6/\omega_0)\Omega_{q_1}^2\eta_1^2\ ]e^{2i(k_1 z-\omega_0 t)} \quad (12)$$

where

$$\delta_5=\delta_1+\left(c\left(4\omega_0^2-\omega_c^2\right)/2\right)\left\{i(\omega_0-i\omega_c)\omega_0\right.$$

$$\left.-(\omega_0+i\omega_c)\omega_c\ \right\}$$

$$\delta_6=\left(\omega_0^2+\omega_c^2\right)\left(4\omega_0^2-\omega_c^2\right)-\omega_0\delta_2$$

The second and third term of the current density equation at the second harmonic arises via. (i) transverse plasma electron velocity oscillation at second harmonic frequency and (ii) coupling of the electron density oscillation of the fundamental electron quiver velocity with oscillation at the fundamental frequency. The latter contributionis attributed to the external magnetic field and provide the source of generation of second harmonic radiation.

The wave equation governing the propagation of the laser pulse through plasma is given by

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E}=\frac{4\pi}{c^2}\frac{\partial\vec{J}}{\partial t}. \quad (13)$$

On placing the fundamental and second harmonic current densities in the above equations leads to fundamental and second harmnioc dispersion relation as

$$c^2k_1^2 = \omega_0^2 - \frac{\omega_0^2\omega_p^2}{\left(\omega_0^2-\omega_c^2\right)} + i\frac{2\omega_p^2}{c}\left\{\frac{\left(\omega_0+i\omega_c\right)}{\left(\omega_0^2-\omega_c^2\right)}\Omega_{q_1}\eta_1\right\} \tag{14a}$$

and

$$c^2k_2^2 = 4\omega_0^2 - \frac{4\omega_0^2\omega_p^2}{\left(4\omega_0^2-\omega_c^2\right)} + i\frac{4\omega_p^2}{c}\left\{\frac{\left(2\omega_0+i\omega_c\right)}{\left(4\omega_0^2-\omega_c^2\right)}\Omega_{q_2}\eta_2\right\} \tag{14b}$$

In the absence of magnetic field $(\omega_c = 0)$ and the quantum contributions $(\hbar = 0)$, Eqn. (14) reduce to the well-known linear dispersion relations for the fundamental and second harmonic waves propagating in plasma [21]. Here, wave refractive indices corresponding to the fundamental and second harmonic are $\mu_1(= ck_1/\omega_0)$ and $\mu_2(= ck_2/2\omega_0)$ respectively.

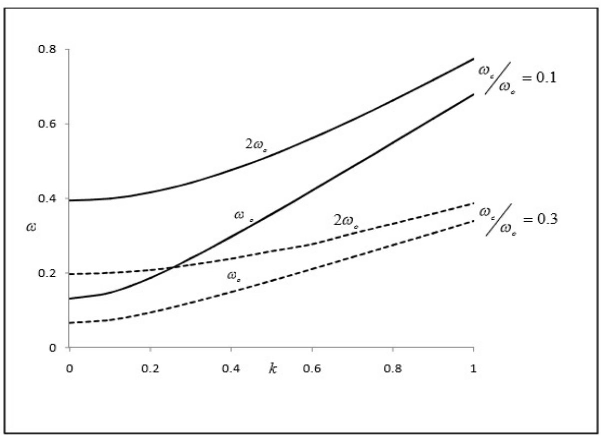

Fig 1 (Variation of (for fundamental frequency ω_0 and second harmonic frequency $2\omega_0$) with k for $n_0 = 10^{28}\,m^{-3}$, $\omega_c/\omega_0 = 0.1$ and 0.3 .The dashed line and solid line are for fundamental and second harmonic frequency respectively.

3. Second harmonic generation

The effect of transverse magnetic fields on second harmonic generation in intense laser plasma interactions is investigated. Possibly the longitudinal motion of electrons is coupled with the transverse motion via the magnetic fields, which lead to generation second order harmonics under normal laser incidence. We consider the distance over which $\partial a_2/\partial z$ changes appreciably is large compared with the wavelength ($\partial^2 a_2(z)/\partial z^2 >> k_2\,\partial a_2(z)/\partial z$), and that a_1 depletes very slightly (with z) so that the quantity a_1^2 can be considered to independent of z. We proceed to obtain the amplitude of the second harmonic field by substituting the current density (Eqn. (12)) into the wave equation (Eqn. (13)) and equating the second order terms, the evolution of second harmonic is governed by

$$\frac{\partial a_2(z)}{\partial z} = \frac{3ia_1^2}{c}\frac{k_1}{k_2}\frac{\omega_p^2\omega_0^2\omega_c\left(\omega_0^2+\omega_c^2\right)}{\left(\omega_0^2-\omega_c^2\right)^2\left(4\omega_0^2-\omega_c^2\right)} - \frac{4\omega_p^2a_1ek_1\eta_1\Omega_{q_1}}{c^3ek_2\left(4\omega_0^2-\omega_c^2\right)\left(\omega_0^2-\omega_c^2\right)^2}\left[\delta_6\eta_1\Omega_{q_1}+\delta_5\quad\right]e^{i\Delta k.z} \tag{15}$$

(15)

$$a_2(z) = \frac{3ia_1^2}{c}\frac{k_1}{k_2}\frac{\omega_p^2\omega_0^2\omega_c\left(\omega_0^2+\omega_c^2\right)}{\left(\omega_0^2-\omega_c^2\right)^2\left(4\omega_0^2-\omega_c^2\right)} - \frac{4\omega_p^2a_1k_1\eta_1\Omega_{q_1}}{c^3k_2\left(4\omega_0^2-\omega_c^2\right)\left(\omega_0^2-\omega_c^2\right)^2}\left[\delta_6\eta_1\Omega_{q_1}+\delta_5\quad\right] \times e^{i\frac{\Delta k.z}{2}\frac{\sin(\Delta k.z/2)}{\Delta k}} \tag{16}$$

Where $\Delta k = k_2 - 2k_1$

The second harmonic conversion efficiency (η) is defined as

$$\eta = \frac{\mu_2}{\mu_1}\frac{\left|a_2^2\right|}{\left|a_1^2\right|} \tag{17}$$

$$\eta = \frac{\omega_p^4k_1}{2c^6k_2\left(\omega_0^2-\omega_c^2\right)^4\left(4\omega_0^2-\omega_c^2\right)^2} \times\left[3ia_1\omega_0^2\omega_c^2\left(\omega_0^2+\omega_c^2\right)+4\eta_1\Omega_{q_1}\left\{\delta_6\eta_1\Omega_{q_1}+\delta_5\right\}\right]$$

$$\times \frac{\sin^2(\Delta k.z/2)}{(\Delta k)^2} \tag{18}$$

Fig. 1, shows the curves of dispersion relation for the frequency $2\omega_0$ and ω_0 with k for $\omega_c/\omega_0 = 0.1$ and 0.3. It is observed that ω (for second harmonic frequency $2\omega_0$ and fundamental frequency ω_0) increase with increase in k but on the other hand frequency decreases with increase in the applied static magnetic field. The dispersion of second harmonic frequency is noticed to be 35.66% less in comparison to the fundamental frequency. Fig 2 shows the curve of conversion efficiency (η) with z for $\omega_p/\omega_0 = \omega_c/\omega_0 = 0.3$ for a laser beam of intensity $10^{17}\,W/cm^2$ and wavelength $1\,\mu m$ $(a_1^2 = 0.02)$ propagating through transversely magnetized plasma. The conversion efficiency increase up to 0.025% for $\omega_c/\omega_0 = 0.3$.

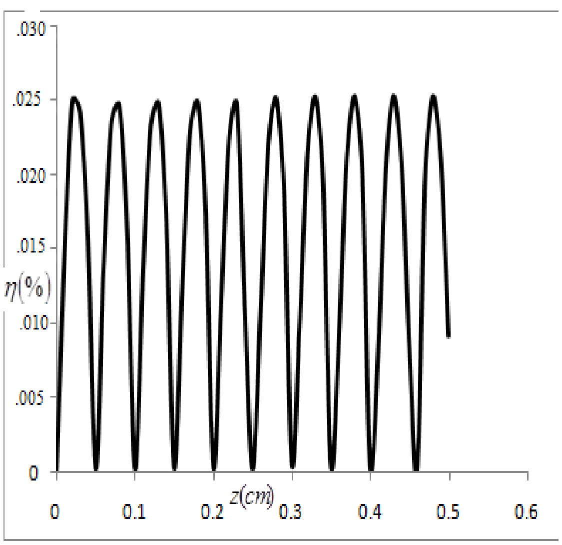

Fig 2: (Variation of conversion efficiency (η) with z for $\omega_p/\omega_0 = \omega_c/\omega_0 = 0.3$)

4. Discussion

In the present work, we have investigated second harmonic generation for the propagation of linearly polarized laser pulse in uniform transversely magnetized quantum plasma based on the quantum hydrodynamic (QHD) equations. The effect of quantum Bohm potential and Fermi pressure have been taken in the account. It may be noted that the generation of second harmonics in homogeneous plasma could point towards the possibility of the presence of a magnetic field, since second harmonics have so far been generated by the passage of a linearly polarized laser in homogeneous plasma. The amplitude of the second harmonics has been derived and hence its conversion efficiency has been obtained. It is seen that the second harmonic conversion efficiency oscillates as the wave progresses along the z direction. It is found that the maximum conversion efficiency is 0.025% in the quantum plasma. The present study will be of immense use in the study of dense astrophysical environments and in future generation of high density plasma experiments. Laser plasma interaction is important in charged particle acceleration, laser Wakefield phenomenon as well as other related fields. Laser based synchrotron radiation [22], Femtosecond x rays from laser-plasma accelerators [23] are of extreme interest in present scenario. Laser–plasma interactions in the relativistic regime [24] is similarly one such important field of study. Laser matter interaction is thus a matter of great academic and research interest [25]. In all these phenomenon harmonic generation and new modes of oscillation has been observed. The existence of recombination harmonics and positronium harmonics are reported by Salamin [25]. Higher order harmonic generation is one of the few effects like electron laser acceleration, atomic quantum dynamics and associated phenomenon, quantum electro dynamical effects, and nuclear interactions in plasmas and ions. These are among the important phenomenon observed when laser interacts with plasma.

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