LRS Bianchi Type I Universe Filled with Interacting Cold Dark Matter and Holographic Dark Energy

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Abstract

This paper deals with the study of anisotropic and homogeneous locally rotationally symmetric (LRS) Bianchi type I universe filled with interacting dark matter and holographic dark energy. The solutions of the field equations are obtained under the assumption that expansion scalar θ is proportional to shear scalar σ . The State finder diagnostic is applied to the model in order to distinguish between our dark energy model with other existing dark energy models. The physical and geometrical aspects of the models are also discussed.

1. Introduction

The type-Ia supernovae (SNeIa) observation indicates that the Universe is currently not only expanding but also accelerating [1-2]. It is suggested that the Universe is spatially flat and dominated by an exotic component with large negative pressure called dark energy (DE) and dark matter (DM) [3-10]. The Universe consists of about 73% of the dark energy and 23% of dark matter out of the total energy density of the Universe (Wilkinson Microwave Anisotropy Probe (WMAP)).

There are many candidates for dark energy, namely the quintessence scalar field models [11], [12]), tachyon field [13-14]), phantom field [15-17] K-essence [18-20], the dark energy models including Chaplygin gas [21, 22], quintom [23-24]), and so on.

Among the many different approaches to describe the dark cosmological sector, so called holographic dark energy models, have received considerable attention [25-27]. According to the holographic principle, the number of degrees of freedom in a bounded system should be finite and related to the area of its boundary [28]. Based on this principle, a field theoretical relation between a short distance (ultraviolet) cut-off and a long distance (infrared) cut-off was established [29]. This relation ensures that the energy in a box of size L does not exceed the energy of a black hole of the same size.

Applied to the dynamics of the Universe, L has to be a cosmological length scale. Different choices of this cut-off scale result in different DE models. If one identifies L with the Hubble radius H^{-1}, the resulting DE density corresponding to the ultra-violet cut-off will be close to the observed effective cosmological constant, [25- 27]) have subsequently shown the possibilities of the particle and the event horizons as the IR cut-off length and found that apparently only a future event horizon cut-off can give a viable DE model. However, afterwards it was recognized, that a cut-off at the Hubble scale may well result in a realistic cosmological dynamics provided only, that an interaction in the dark sector is admitted [29]). Holographic DE model have been tested and constrained by various astronomical observations [30-33]. A special class are models in which holographic DE is allowed to interact with DM [29] and [34-55]. Recently, Sarkar [56-58] has studied non-interacting holographic dark energy with linearly varying deceleration parameter in Bianchi type I and V universes and interacting holographic dark energy in Bianchi type-II. Adhav el al. [59] has studied interacting and holographic dark energy model in Bianchi Type I universe. Besides, some interacting models are discussed in many works because these models can help to understand or alleviate the coincidence problem by considering the possible interaction between dark energy and cold dark matter due to the unknown nature of dark energy and dark matter. In addition, the proposal of interacting dark energy is compatible with the current observations such as the SNIa and CMB data (Guo, Ohta and Tsujikawa [60]).

The anisotropy plays a significant role in the early stage of evolution of the Universe and hence the study of anisotropic and homogeneous cosmological models becomes important. In the present paper, we consider LRS Bianchi type I universe filled with interacting dark matter and holographic dark energy. The geometrical and physical aspects of the models are also studied.

The physical parameters that are of cosmological importance for Bianchi type I space-time are:

The mean Hubble parameter

$$H=\frac{\dot{a}}{a}=\frac{1}{3}\frac{\dot{V}}{V} \tag{1}$$

The deceleration parameter

$$q=-\frac{\ddot{a}a}{\dot{a}^2}=\frac{d}{dt}\left(\frac{1}{H}\right)-1 \tag{2}$$

The mean anisotropy parameter

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2 \tag{3}$$

where $a=(AB^2)^{1/3}=V^{1/3}$ is the mean scale factor and $H_1=\frac{\dot{A}}{A}$, $H_2=H_3=\frac{\dot{B}}{B}$ are the directional Hubble parameters in the directions of x, y, z axes, respectively.

2. Metric and field equations

The LRS Bianchi-Type-I line element can be written as

$$ds^2=dt^2-A^2dx^2-B^2(dy^2+dz^2) \tag{4}$$

where A and B are the scale factors and functions of the cosmic time t only.

The Einstein's field equations are ($8\pi G=1$ and $c=1$)

$$R_{ij}-\frac{1}{2}g_{ij}R=-({}^{m}T_{ij}+{}^{\Lambda}T_{ij}) \tag{5}$$

where ${}^{m}T_{ij}=\rho_m u_i u_j$ and

$${}^{\Lambda}T_{ij}=(\rho_\Lambda+p_\Lambda)u_iu_j+g_{ij}p_\Lambda \tag{6}$$

are matter tensor for cold dark matter (pressure-less i.e., $w_m=0$) and holographic dark energy. Here ρ_m is the energy density of dark matter and ρ_Λ and p_Λ are the energy density and pressure of holographic dark energy.

The Einstein's field equations (Eqn. (5)) for metric (Eqn. (4)) with the help of Eqn. (6) can be written as

$$\frac{2\ddot{B}}{B}+\frac{\dot{B}^2}{B^2}=-p_\Lambda \tag{7}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}=-p_\Lambda \tag{8}$$

$$\frac{\dot{B}^2}{B^2}+\frac{2\dot{A}\dot{B}}{AB}=\rho_\Lambda+\rho_m \tag{9}$$

where overhead dot (˙) represents the derivative with respect to time *t*.

Furthermore, we assume that both components do not conserve separately but interact with each other in such a manner that the balance equations take the form

$$\dot{\rho}_m+\left(\frac{\dot{V}}{V}\right)\rho_m=Q \tag{10}$$

$$\dot{\rho}_\Lambda+\left(\frac{\dot{V}}{V}\right)(1+w_\Lambda)\rho_\Lambda=-Q \tag{11}$$

where $w_\Lambda=p_\Lambda/\rho_\Lambda$ is the equation of state parameter for holographic dark energy and $Q>0$ measures the strength of the interaction. Models featuring an interaction matter-dark energy were introduced in [61,62] and [63], and first used alongside the holographic dark energy by [64]. Although the assumption of a coupling between both components implies the introduction of an additional phenomenological function *Q*, a description that admits interactions is certainly more general than otherwise. Further, there is no known symmetry that would suppress such interaction and arguments in favor of interacting models have been put forward recently in [65]). A vanishing Q implies that matter and dark energy remain separately conserved. In view of continuity equations, the interaction between DE and DM must be a function of the energy density multiplied by a quantity with units of inverse of time, which can be chosen as the Hubble factor H. There is freedom to choose the form of the energy density, which can be any combination of DE and DM. Thus, the interaction between DE and DM could be expressed phenomena -logically in forms such as in [60] and [66]).

$$Q=3b^2H\rho_m=b^2\frac{\dot{V}}{V}\rho_m \tag{12}$$

where b^2 is the coupling constant. Cai and Wang [67] have taken same relation for interacting phantom dark energy and dark matter in order to avoid the coincidence problem.

From equations (10) and (12), we get the energy density of dark matter as

$$\rho_m=\rho_0V^{(b^2-1)} \tag{13}$$

where $\rho_0>0$ is a real constant of integration.

Using equations (12) and (13), we get the interacting term as

$$Q=3\rho_0b^2HV^{(b^2-1)}. \tag{14}$$

3. Cosmological solutions

In order to obtain exact solutions of the field equations (7) - (9), we assume that the expansion scalar (θ) in the model is proportional to the shear scalar (σ). This condition leads to

$$A = B^n \tag{15}$$

where $n > 1$ is constant.
Subtracting Eqn. (7) from Eqn. (8), we get

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{B}^2}{B^2} = 0 \tag{16}$$

Using Eqn. (15) in the Eqn. (16) and then on integrating, we obtain the value of scale factors as

$$A = \left[\frac{n+2}{n}\right]^{\frac{n}{n+2}} (c_1 t + c_2)^{\frac{n}{n+2}} \tag{17}$$

$$B = \left[\frac{n+2}{n}\right]^{\frac{1}{n+2}} (c_1 t + c_2)^{\frac{1}{n+2}} \tag{18}$$

where $c_1 > 0$ and c_2 are real constants of integration.
The volume scale factor V is given by

$$V = AB^2 \tag{19}$$

$$V = \left[\frac{n+2}{n}\right] (c_1 t + c_2) \tag{20}$$

Using Eqn. (20) in Eqns. (13) and (14), we get

$$\rho_m = \rho_0 \left[\frac{n+2}{n}\right]^{(b^2-1)} (c_1 t + c_2)^{(b^2-1)} \tag{21}$$

$$Q = b^2 c_1 \rho_0 \left[\frac{n+2}{n}\right]^{(b^2-1)} (c_1 t + c_2)^{(b^2-2)} \tag{22}$$

Using Eqns. (17), (18) and (21) in Eqn. (9), we obtain the energy density of holographic dark energy as

$$\rho_\Lambda = \frac{c_1^2(1+2n)}{(n+2)^2 (c_1 t + c_2)^2} - \rho_0 \left[\frac{n+2}{n}\right]^{(b^2-1)} (c_1 t + c_2)^{(b^2-1)} \tag{23}$$

Using Eqns. (17), (18), (21) and (23) in the linear combination of Eqns. (7- 9), we obtain the pressure of holographic dark energy as

$$p_\Lambda = \frac{c_1^2(2n+1)}{(n+2)^2 (c_1 t + c_2)^2} \tag{24}$$

The EoS parameter of holographic dark energy is given by

$$w_\Lambda = \frac{p_\Lambda}{\rho_\Lambda} = \frac{c_1^2 \dfrac{(2n+1)}{(n+2)^2 (c_1 t + c_2)^2}}{\dfrac{c_1^2(1+2n)}{(n+2)^2 (c_1 t + c_2)^2} - \rho_0 \left[\dfrac{n+2}{n}\right]^{(b^2-1)} (c_1 t + c_2)^{(b^2-1)}} \tag{25}$$

Using Eqns. (17- 18) in Eqns. (1), (2) and (3), we get the mean Hubble parameter, deceleration parameter and mean anisotropy parameter of expansion as

$$H = \frac{c_1}{3(c_1 t + c_2)} \tag{26}$$

$$\Delta = \frac{2(n-1)^2}{(n+2)^2} \tag{27}$$

The coincidence parameter $\bar{r} = \rho_m / \rho_\Lambda$, i.e., the ratio of dark matter energy density to the dark energy density is given by

$$\bar{r} = \frac{\rho_0 \left[\dfrac{n+2}{n}\right]^{(b^2-1)} (c_1 t + c_2)^{(b^2-1)}}{\dfrac{c_1^2(1+2n)}{(n+2)^2 (c_1 t + c_2)^2} - \rho_0 \left[\dfrac{n+2}{n}\right]^{(b^2-1)} (c_1 t + c_2)^{(b^2-1)}} . \tag{28}$$

5. State-finder diagnostic

In order to be able to differentiate between those competing cosmological scenarios involving DE, a sensitive and robust diagnostic for DE models is a must. For this purpose, a diagnostic proposal that makes use of parameter pair {*r*, *s*}, the so-called "state-finder", was introduced by Sahni et al. [68]. The state-finder probes the expansion dynamics of the Universe through higher derivatives of the expansion factor $\dddot{a}$ and is a natural companion to the deceleration parameter, which depends upon $\ddot{a}$. The state-finder pair {*r*, *s*} is defined as follows

$$r = \frac{\dddot{a}}{aH^3} \text{ and } s = \frac{r-1}{3(q-1/2)}$$

The state-finder is a 'geometrical' diagnostic in the sense that it depends upon the expansion factor and hence upon the metric describing space-time. Trajectories in the *s-r* plane corresponding to different cosmological models exhibit qualitatively different behaviors. The spatially flat Λ CDM scenario corresponds to a fixed point in the diagram $\{s, r\}\big|_{\Lambda CDM} = \{0, 1\}$. Departure of a given DE model from this fixed point provides a good way of establishing the 'distance' of this model from Λ CDM [59]. The state-finder can successfully differentiate between a wide variety of DE models including the cosmological constant, quintessence, the Chaplygin gas, brane world models and interacting DE models [60], [65 - 67].

$$r = 10 \text{ and } s = \frac{r}{5}.$$

6. Discussion and conclusion

In this paper we have studied the anisotropic and homogeneous LRS Bianchi type I universe filled with interacting dark matter and holographic dark energy. The solutions of the field equations are obtained under the assumption that expansion scalar θ is proportional to shear scalar σ. In the holographic interacting dark energy model by employing the apparent horizon as the IR cut off [69], it was argued that an equation of state of dark energy $w_\Lambda < 0$ is necessarily accompanied by the decay of the dark energy component into pressure-less dark matter ($b^2 > 0$). Also, it is observed that the model remains anisotropic throughout the evolution of the Universe. Fig. 1 shows the evolution of EoS parameter w_Λ with cosmic time *t*. It is observed that w_Λ starts with a very small negative value and enters into quintessence region $(-1 < w < -1/3)$ and attains the value $w_\Lambda = -1$ after some finite *t*, i.e., the model approaches to $\wedge$CDM model after some finite *t*. The available data sets in cosmology, especially the SNeIa data [70 - 72], the SDSS data [73], and the three year WMAP data [4] indicate that the $\wedge$CDM model or the model that reduces to $\wedge$CDM serves as a standard model in cosmology, an excellent model to describe the cosmological evolution. Soon after attaining $w_\Lambda = -1$, w_Λ diverges in the phantom region ($w_\Lambda < -1$). The State-finder diagnostic is applied to the models in order to distinguish between our dark energy models with other existing dark energy models. Fig. 2 shows the evolving trajectory in the *s-r* plane for corresponding models is quite different from those of other DE models. We hope that the future high precision observations will be capable of determining these state-finder parameters and consequently explore the nature of DE.

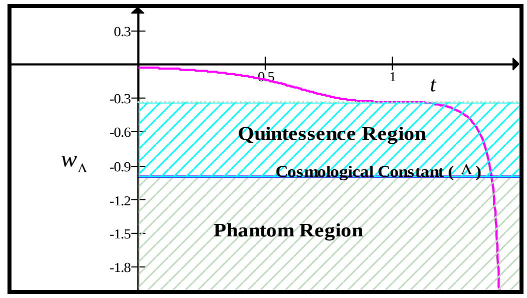

Fig.1:Evolution of EoS parameter (w_Λ).

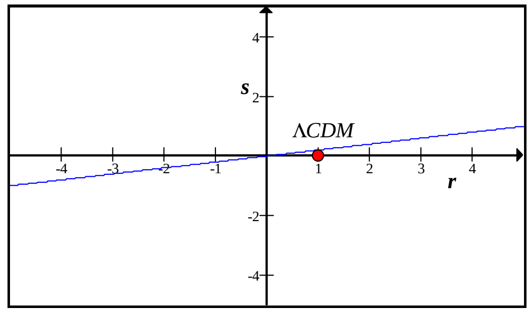

Fig.2- State-finder parameters *s* versus *r*

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