Magnetized anisotropic modified holographic Ricci dark energy viscous cosmological model

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Abstract: A bulk viscous magnetized locally rotationally symmetric (LRS) Bianchi type I cosmological model has been studied in general relativity. To find exact solutions of Einstein's field equations we use modified holographic ricci dark energy density in presence of bulk viscosity and magnetic field. The coefficient of bulk viscosity ζ is taken as a quadratic function of Hubble parameter H which is of the form $\zeta = \zeta_0 + \zeta_1 H + \zeta_2 H^2$ where ζ_0, ζ_1 and ζ_2 are constants. Also, we have considered hybrid expansion law to find the solutions. The physical and geometrical aspects of the cosmological model have been discussed and it is found that the results are in good agreement with the present-day observational facts.

1. Introduction

Supernova observations (SN Ia) established the fact that the expansion of the universe [1-3] is caused by a kind of an exotic energy called dark energy (DE) with large negative pressure. Planck cosmological results [4] and WMAP measurements conclude that the universe consists of about dark energy 68.3%, dark matter 26.8% of the total energy of the universe and the rest 4.9% energy is baryonic matter. In modern cosmology, the dark energy plays an important role to establish the concept of accelerating universe. But the nature of dark energy is still unknown and is a mystery and researchers have suggested some candidates to describe it. The cosmological constant Λ satisfying the equation of state $\omega = -1$ is the simplest candidate for dark energy. But it has the problems of 'fine-tuning' and 'cosmic coincidence'. Other candidates for dark energy are suggested viz. quintessence, phantom, quintom, tachyon, k-essence, dilation, holographic, pilgrim dark energy etc. Considerable research works have been done and many different models related to dark energy have been suggested [5, 6].

The study of holographic dark energy (HDE) model now-a-days becomes important to describe the phase transition of the universe. The HDE is based on holographic principle [7, 8]. The holographic principle was first proposed by `t Hooft [7]. It occupies an important role in black hole and string theory. According to this principle, the entropy of the system scales not with its volume, but also its surface area (L^2). The energy density of Holographic DE is $\rho_{de} = 3c^2 M_{pl}^2 L^{-2}$where L is the infrared (IR) cut off radius, $M_{pl}^2 = 1/8\pi G$ is the Planck mass and `$c$` is constant [9]. Gao et al. [10] suggested that the dark energy density may be inversely proportional to the area of the event horizon of the Universe. In this model the future event horizon is replaced by the inverse of the Ricci scalar curvature. This model is known as Ricci Dark Energy model. Granda et al. [11, 12] proposed a MRDE model where the density of DE is a function of Hubble parameter $H$and its derivative with respect to cosmic time t. The mathematical expression of this model is $\rho_{de} = 3M_{pl}^2(\eta_1 H^2 + \eta_2 \dot{H})$. Chen and Jing [13] later modified this model where the density of DE contains the second order derivative of H with respect to t. The mathematical expression is $\rho_H = 3M_{pl}^2(\eta_1 H^2 + \eta_2 \dot{H} + \eta_3 \ddot{H} H^{-1})$.

The distribution of galaxies in the universe is described by the matter distribution governed by perfect fluid. The decoupling of neutrinos during the radiation era and the separation of radiation and matter during the recombination era, give rise to viscous effects. Weinberg [14] presented the role of viscosity in cosmology. The coefficient of viscosity is known to decrease as the universe expands. Misner [15, 16] investigated the effect of viscosity during the evolution of the universe. Padmanabhan and Chitre [17] found that the presence of bulk viscosity leads to inflationary-like solutions in general relativity. The concept of bulk viscosity ς introduces dissipation by only redefining the effective pressure $\bar{p}$ where $\bar{p} = p - 3\zeta H$. The coefficient of bulk viscosity ζ is taken to be quadratic function of Hubble parameter H [17] as $\zeta = \zeta_0 + \zeta_1 H + \zeta_2 H^2$. The existence of magnetic field in galactic and intergalactic space plays a very important role and anisotropic magnetic field models have contributions in the evolution of galaxies and stellar objects. Zeldovich et al. [18], Harrison [19] investigated the magnetic field in various cosmological models. Asseo and Sol [20] and Madsen [21] have discussed the primordial magnetic fields. Melvin [22] suggested, in the cosmological solution for dust and electromagnetic fields during the evolution of the universe the matter was in highly ionized state and smoothly coupled with electromagnetic and consequently formed a neutral matter as a result of expansion of the universe. Singh and Singh [23] have discussed the nature of the classical potential for viscous fluid with and without magnetic field. Das and Sultana [24] studied magnetized anisotropic ghost dark energy cosmological models in General Relativity. The magnetized dark energy responsible for the present-day accelerating phase of the universe has been observed by Yadav et al. [25]. Recently Singh et al. [26] have presented a viscous cosmological model in MHRDE where it is observed that the bulk viscosity acts very significant position in the expansion history of the universe. In this paper our aim is to investigate the nature of viscous anisotropic modified holographic dark energy model in presence of magnetic field.

The paper is organised as follows: In Section 2, the relevant field equation with MHRDE viscous cosmological model in GR is presented. In, Section 3, isotropization and solution of field equations are obtained. In Section 4, the graphical discussions of the various parameters verses cosmic time t is presented. In Section 5, a brief conclusion is given.

2. The Metric and Field Equations

We consider the homogenous and anisotropic space-time described by Bianchi type I in the metric form as

$$ds^2 = dt^2 - P^2 dx^2 - Q^2(dy^2 + dz^2) \tag{1}$$

where $P$and $Q$are functions of cosmic time t only.

The Einstein`s field equations with magnetic field are

$$R_{ij} - \frac{1}{2} g_{ij} R = -\left(T_{ij} + \bar{T}_{ij}\right) \tag{2}$$

Where, R_{ij} is the Ricci tensor, R is the Ricci scalar and $T_{ij}$and $\bar{T}_{ij}$ are the energy momentum tensors for matter and MHRDE respectively.

The energy momentum tensor for matter is

$$T^i{}_j = diag[\rho, 0,0,0] \tag{3}$$

where ρ is the energy density of matter.

The energy momentum tensor for MHRDE is as follows:

$$\bar{T}^i{}_j = diag\left[\rho_H + \rho_B, -\bar{p}_{H_x} + \rho_B, -\bar{p}_{H_y} - \rho_B, -\bar{p}_{H_z} - \rho_B\right] \tag{4}$$

where ρ_H is the energy density of MHRDE, $\bar{p}_{H_x}, \bar{p}_{H_y}$and $\bar{p}_{H_z}$are the effective pressures in the directions of x, y and z axes respectively and $\omega_x = \omega_H, \omega_y = \omega_H + \delta$ and $\omega_z = \omega_H + \delta$ are the directional EOS parameters of the MHRDE on x, y and z axes respectively and ρ_B stands for the energy density of magnetic field B. The skewness parameter δ is the deviations from ω_H in the directions of y and z. Here ω_H and δ need not be constants and can be functions of cosmic time t only.

King & Coles [27] used the magnetic perfect fluid energy–momentum tensor to discuss the effects of magnetic flux on the evolution of the Universe. Jacobs [28] studied the impact of a regular, early magnetic flux on Bianchi type-I cosmological model. We assume that the universe is filled with matter and magnetized modified holographic ricci dark energy fluid. A cosmological model which contains a global magnetic field is necessarily anisotropic since the magnetic field vector specifies a preferred spatial direction. In this paper, we considered the electromagnetic field along x-axis ($B = B_x$) only.

The Einstein`s field equation (2) for the space-time (1) takes the form

$$2\frac{\dot{P}\dot{Q}}{PQ} + \frac{\dot{Q}^2}{Q^2} = \rho + \rho_H + \rho_B \tag{5}$$

$$2\frac{\ddot{Q}}{Q}+\frac{\dot{Q}^2}{Q^2}=-\omega_H\rho_H+\rho_B+3H\zeta$$
(6)

$$\frac{\ddot{P}}{P}+\frac{\ddot{Q}}{Q}+\frac{\dot{P}\dot{Q}}{PQ}=-(\omega_H+\delta)\rho_H-\rho_B+3H\zeta$$
(7)

The energy conservation equation $(T^{ij}+\bar{T}^{ij})_{;j}=0$ can be obtained as

$$\dot{\rho}+\left(\frac{\dot{P}}{P}+2\frac{\dot{Q}}{Q}\right)(\rho-3H\zeta)+\dot{\rho}_H+(1+\omega_H)\rho_H\left(\frac{\dot{P}}{P}+\frac{2\dot{Q}}{Q}\right)+2\frac{\dot{Q}}{Q}\delta\rho_H=0$$
(8)

$$\rho_B=\frac{I}{Q^4}$$
(9)

Where, I is a constant.

Where, an overhead dot $(.)$ denote differentiation w.r.to cosmic time t.

3. Solutions of Field Equations

The spatial volume V is defined as

$$R^3=V=PQ^2$$
(10)

Where. R is the average scale factor.

The average Hubble parameter H is defined as

$$H=\frac{\dot{V}}{3V}=\frac{1}{3}\left(\frac{\dot{P}}{P}+\frac{2\dot{Q}}{Q}\right)=\frac{1}{3}\left(H_x+2H_y\right)$$
(11)

where $H_x=\frac{\dot{P}}{P}$ and $H_y=H_z=\frac{\dot{Q}}{Q}$ are the directional Hubble parameters in the x,y and z axes respectively.

The deceleration parameter q is defined as

$$q=-\frac{R\ddot{R}}{\dot{R}^2}$$
(12)

Equations (5)-(8) are four field equations with seven unknowns $P,Q,\rho,\rho_H,\omega_H,\zeta,$ and δ. So, in order to solve them completely, we need the following three extra relations:

(i) Chen and Jing [13] suggested the MHRDE density as

$$\rho_H=3\left(\eta_1H^2+\eta_2\dot{H}+\eta_3\ddot{H}H^{-1}\right)$$
(13)

Where, η_1,η_2 and η_3 are constants and $M_{pl}^2=8\pi G=1$.

(ii) Akarsu et al. [29] proposed the average scale factor $R(t)$ as a mixture of power law and exponential law as

$$R(t)=R_0t^{h_1}e^{h_2t} \tag{14}$$

where h_1 and h_2 are non-negative constants and R_0 represents the present value of scale factor.

The relation (14) yields an exponential law when $h_1=0$ and the power law when $h_2=0$. The relation (14) is a mixture of power and exponential law which is usually called Hybrid Expansion Law (HEL).

(iii) The bulk viscosity ζ according to [17] is assumed as

$$\zeta=\zeta_0+\zeta_1H+\zeta_2H^2 \tag{15}$$

Where, ζ_0,ζ_1 and ζ_2 are constants and H is the Hubble parameter.

Equations (6) and (7) yield

$$\boldsymbol{\frac{\dot{P}}{P}-\frac{\dot{Q}}{Q}=\frac{c_1}{V}e^{-\int\frac{\delta\rho_H+\frac{2I}{Q^4}}{\frac{\dot{P}}{P}-\frac{\dot{Q}}{Q}}dt}}$$
(16)

Where, c_1 is a constant of integration.

To solve the Equation (16), we take (according to Adhav [30])

$$\delta\rho_H+\frac{2I}{Q^4}=\frac{\dot{P}}{P}-\frac{\dot{Q}}{Q}$$
(17)

Using Equation (17), Equation (16) takes the form

$$\frac{\dot{P}}{P}-\frac{\dot{Q}}{Q}=\frac{c_1}{V}e^{-t}$$
(18)

Using Equations (10) and (14) in Equation (18), we get

$$\frac{\dot{P}}{P}-\frac{\dot{Q}}{Q}=\frac{c_1}{R_0^3t^{3h_1}e^{h_3t}}$$
(19)

Where, $h_3=1+3h_2=$ constant.

Integrating Equation (19), we get

$$P = Qc_2\, exp\left[\frac{c_1}{R_0^3}\int \frac{dt}{t^{3h_1}e^{h_3 t}}\right]$$
(20)

where c_2 is a constant of integration.

Equation (10), on using equation (14) gives the spatial volume $V$of the model as

$$R^3 = V = PQ^2 = R_0^3 t^{3h_1}e^{3h_2 t}$$
(21)

Equations (20) and (21) yield

$$Q = \frac{R_0 t^{h_1}e^{h_2 t}}{c_2^{\frac{1}{3}}\left[exp\left\{\frac{c_1}{R_0{}^3}\int \frac{dt}{t^{3h_1}e^{h_3 t}}\right\}\right]^{1/3}} \quad (22)$$

$$P = \frac{R_0 t^{h_1}e^{h_2 t}}{c_2{}^{-\frac{2}{3}}\left[exp\left\{\frac{c_1}{R_0{}^3}\int \frac{dt}{t^{3h_1}e^{h_3 t}}\right\}\right]^{-\frac{2}{3}}}$$
(23)

The expressions for directional Hubble parameters, Hubble parameter and Deceleration parameter are given by

$$H_x = \frac{\dot{P}}{P} = \frac{h_1}{t} + h_2 + \frac{2}{3}\frac{c_1}{R_0^3}t^{-3h_1}e^{-h_3 t}$$
(24)

$$H_y = H_z = \frac{\dot{Q}}{Q} = \frac{h_1}{t} + h_2 - \frac{1}{3}\frac{c_1}{R_0^3}t^{-3h_1}e^{-h_3 t}$$
(25)

$$H = \frac{h_1}{t} + h_2$$
(26)

$$q = -1 + \frac{h_1}{(h_1 + h_2 t)^2}$$
(27)

Equation (8) yields the conservation equation for matter and MHRDE as

$$\dot{\rho} + \left(\frac{\dot{P}}{P} + 2\frac{\dot{Q}}{Q}\right)\rho = 9H^2\zeta$$
(28)

$$\dot{\rho}_H + (1+\omega_H)\rho_H\left(\frac{\dot{P}}{P} + 2\frac{\dot{Q}}{Q}\right) + 2\frac{\dot{Q}}{Q}\delta\rho_H = 0$$
(29)

Equation (28) implies $\dot{\rho} + 3H\rho = 9H^2\zeta$

$$\rho = 9t^{-3h_1}e^{-3h_2 t}\int t^{3h_1}e^{3h_2 t}\left[\varsigma_0\left(\frac{h_1}{t}+h_2\right)^2 + \varsigma_1\left(\frac{h_1}{t}+h_2\right)^3 + \varsigma_2\left(\frac{h_1}{t}+h_2\right)^4\right]dt + c_3 t^{-3h_1}e^{-3h_2 t}$$
(30)

where c_3 is an integrating constant.

Equation (13) and Equation (26) yield the density (MHRDE) as

$$\rho_H = 3t^{-2}[\eta_1(h_1 + h_2 t)^2 - h_1\eta_2 + 2h_1\eta_3(h_1 + h_2 t)^{-1}]$$
(31)

The skewness parameter δ is obtained from equation (17) with the use of equations (24) and (25) as

$$\delta\rho_H = \frac{c_1}{R_0^3 t^{3h_1}e^{h_3 t}} - \frac{2I}{Q^4}$$
(32)

Equation (29) with the use of Equations (24), (25) and (32) yield the expression for ω_H as

$$3H\omega_H\rho_H = \frac{6\eta_1 h_1}{t^2}\left(\frac{h_1}{t} + h_2\right) - \frac{6h_1\eta_2}{t^3} + \frac{6h_1\eta_3(2th_1 + 3t^2h_2)}{t^6\left(\frac{h_1}{t}+h_2\right)^2} - 3H\rho_H - \frac{2\dot{Q}}{Q}\left(\frac{c_1}{R_0^3 t^{3h_1}e^{h_3 t}} - \frac{2I}{Q^4}\right)$$
(33)

The anisotropy parameter $\overline{A_n}$ is given by

$$\overline{A_n} = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{2c_1^2}{9H^2R_0^6}t^{-6h_1}e^{-2h_3 t}$$
(34)

The co-efficient of bulk viscosity $\zeta$is taken as

$$\zeta = \zeta_0 + \zeta_1 H + \zeta_2 H^2 = \zeta_0 + \zeta_1\left(\frac{h_1}{t} + h_2\right) + \zeta_2\left(\frac{h_1}{t} + h_2\right)^2$$
(35)

The matter energy density Ω and the MHRDE density Ω_H are defined as

$$\Omega = \frac{\rho}{3H^2}\ , \Omega_H = \frac{\rho_H}{3H^2}$$
(36)

The total energy density parameter Ω_T is defined as

$$\Omega_T = \Omega + \Omega_H = (\rho + \rho_H)/3H^2$$
(37)

The energy density for magnetic field is obtained as

$$\rho_B = \frac{I}{Q^4} = I\frac{R_0^{-4}t^{-4h_1}e^{-4h_2 t}}{c_2^{\frac{-4}{3}}\left\{exp\left[\frac{c_1}{R_0^3}\int\frac{dt}{t^{3h_1}e^{h_3 t}}\right]\right\}^{\frac{-4}{3}}}$$
(38)

3.1 Cosmic jerk parameter

Cosmic jerk parameter is defined as the third order derivative of the average scale factor w.r.to the cosmic time. It is according to reference [31] given by

$$j(t) = \frac{1}{H^3}\frac{\dddot{R}}{R} = q + 2q^2 - \frac{\dot{q}}{H} \quad (39)$$

Using Equations (26) and (27) in equation (39), we get the expression of cosmic jerk parameter as

$$j(t) = 1 - 3h_1(h_1 + h_2 t)^{-2} + 2{h_1}^2(h_1 + h_2 t)^{-4} + \frac{2h_1 h_2 t}{(h_1 + h_2 t)^4} \quad (40)$$

The transition from the decelerating to the accelerating phase of the universe is due to a cosmic jerk parameter and it occurs for different models with a positive value of the jerk parameter and the negative value of the deceleration parameter [31-33]

4. Graphical Discussions

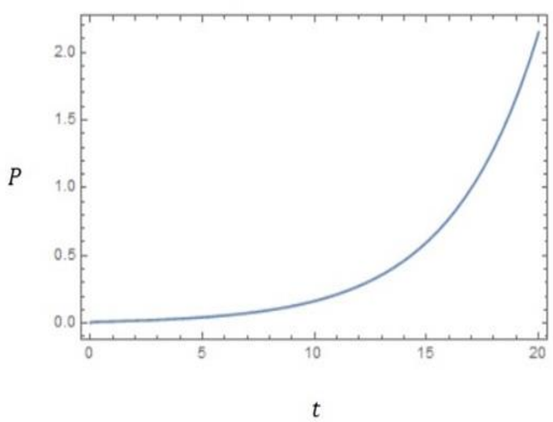

Fig.1: The figure shows the plot of P verses cosmic time t with $R_0 = 1, h_1 = 0.1, h_2 = 0.25, h_3 = 1.75, c_2 = 0.001, c_1 = 0.12$.

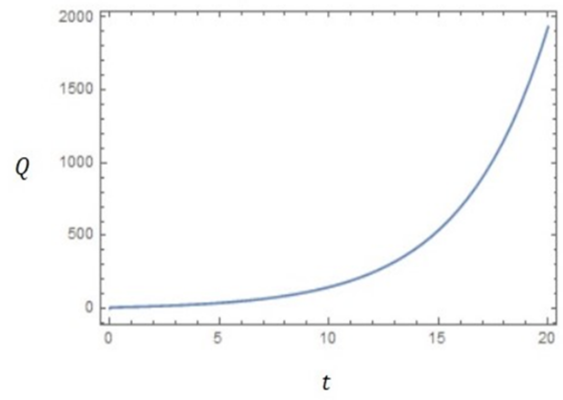

Fig.2: The figure shows the plot of Q verses cosmic time t with $R_0 = 1, h_1 = 0.1, h_2 = 0.25, h_3 = 1.75, c_2 = 0.001, c_1 = 0.12$.

From Fig.1 and Fig.2, it is observed that P and Q increases with time.

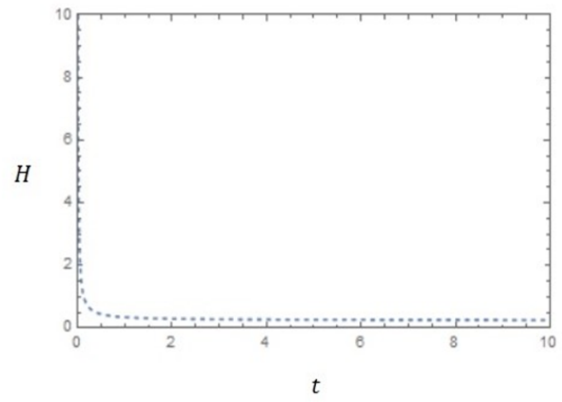

Fig.3: The figure shows the plot of H verses cosmic time t with $h_1 = 0.1, h_2 = 0.25$

From this figure it is seen that H is a decreasing function of t and vanishes for large values of t.

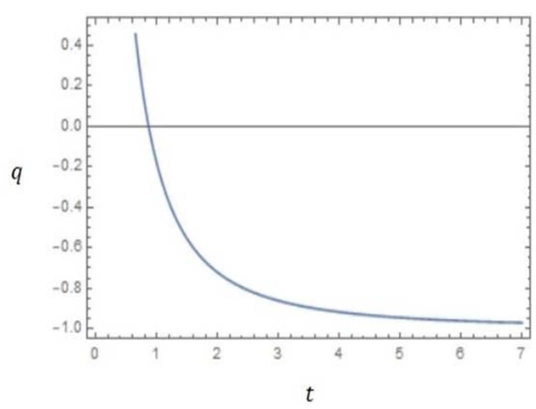

Fig.4: The figure shows the plot of q verses cosmic time t with $h_1 = 0.1, h_2 = 0.25$. From this figure it is seen that q is a decreasing function of t and tends to -1 for large values of t. Deceleration parameter (q) is positive at early stage of the universe and is negative at later age of the universe. This implies that the universe undergoes transition from the decelerating to accelerating phase.

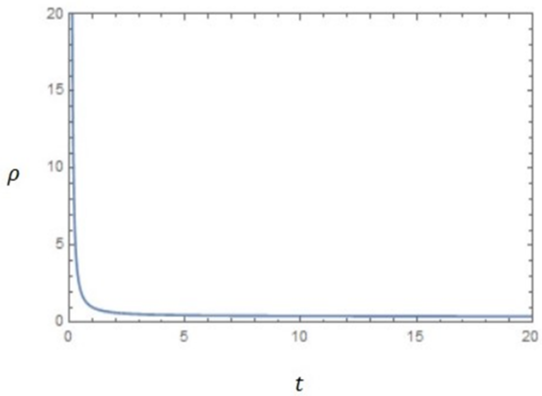

Fig.5: The figure shows the plot of ρ verses cosmic time t with $h_1 = 0.1, h_2 = 0.25, \zeta_0 = 0.01, \zeta_1 = 0.1, \zeta_2 = 0.001, c_3 = 0.1$. From this figure it is seen that ρ is a decreasing function of t and tends to zero at late times.

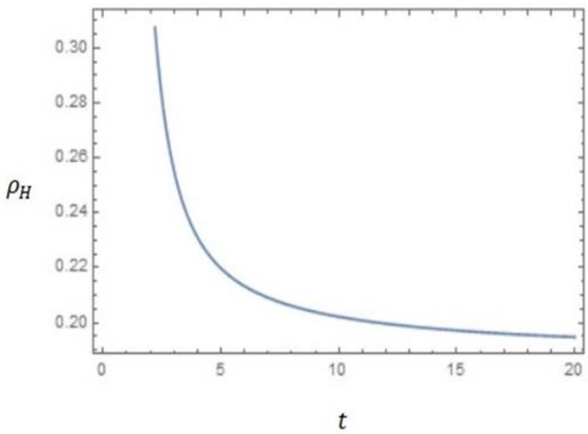

Fig.6: The figure shows the plot of ρ_H verses cosmic time t with $h_1 = 0.1, h_2 = 0.25, \eta_1 = 1, \eta_2 = 0.5, \eta_3 = 0.4$. From this figure it is seen that ρ_H is a decreasing function of t and tends to small value at late times.

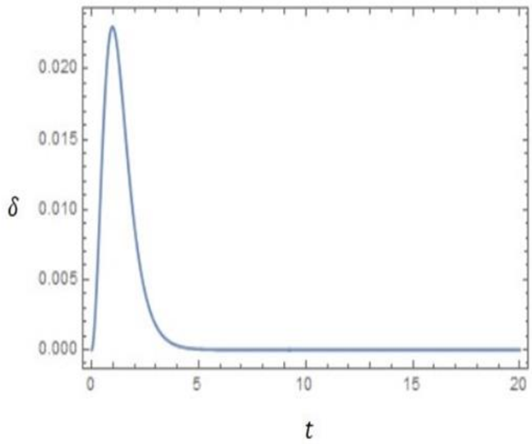

Fig.7: The figure shows the plot of δ verses cosmic time t with. $h_1 = 0.1, h_2 = 0.25, \eta_1 = 1, \eta_2 = 0.5, \eta_3 = 0.4, R_0 = 1, c_1 = 0.12, I = 1$. From this figure it is seen that δ increases sharply at early stage of the universe and then decreases and ultimately tends to zero at late times.

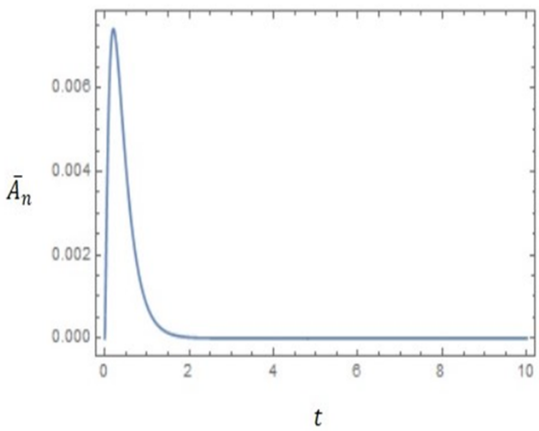

Fig.8: The figure shows the plot of $\overline{A_n}$ verses cosmic time t with. $h_1 = 0.1, h_2 = 0.25, h_3 = 1.75, R_0 = 1, c_1 = 0.12$. From this figure it is seen that $\overline{A_n}$ increases sharply at early stage of the universe and then decreases and ultimately tends to zero at late times. It is evident that at the late times the universe becomes isotropic.

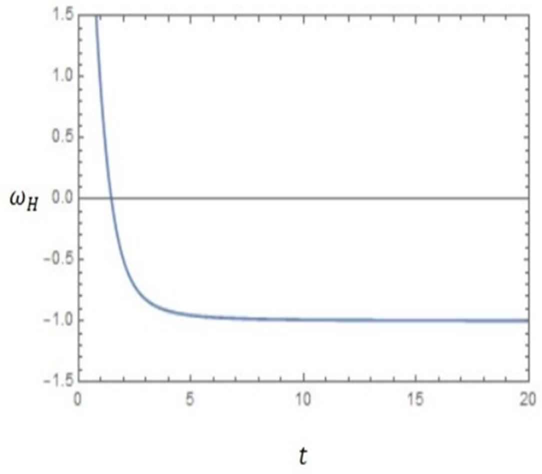

Fig.9: The figure shows the plot of ω_H verses cosmic time t with $h_1 = 0.1, h_2 = 0.25, \eta_1 = 1, \eta_2 = 0.5, \eta_3 = 0.4, R_0 = 1, c_1 = 0.12, I = 1$. From this figure it is seen that ω_H tends to -1 at late times and it behaves like a cosmological constant.

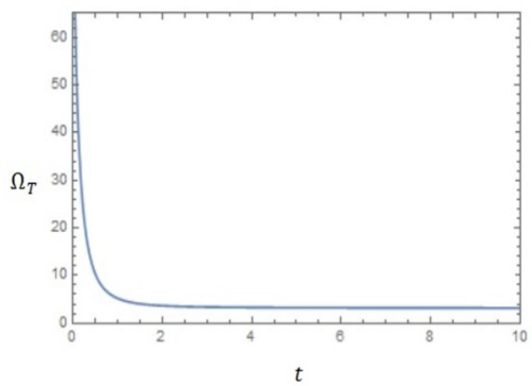

Fig.10: The figure shows the plot of Ω_T verses cosmic time t with $h_1 = 0.1, h_2 = 0.25, \eta_1 = 1, \eta_2 = 0.5, \eta_3 = 0.4, c_3 = 0.1$. From this figure it is seen that as the universe expands, Ω_T tends to 1 at late times and thus the universe becomes spatially homogenous, isotropic and flat.

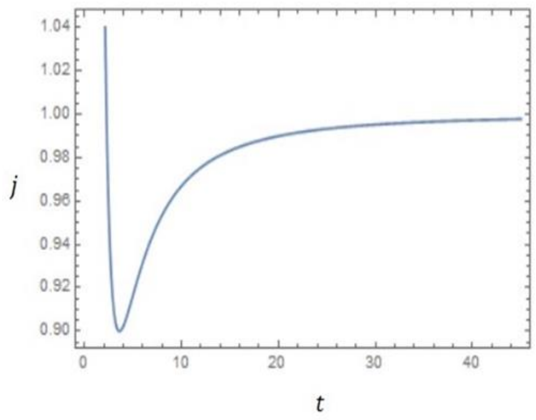

Fig.11: The figure shows the plot of j verses cosmic time t with $h_1 = 0.1, h_2 = 0.25$. From this figure it is seen that j tends to 1 at late times and is positive throughout the entire age of the universe.

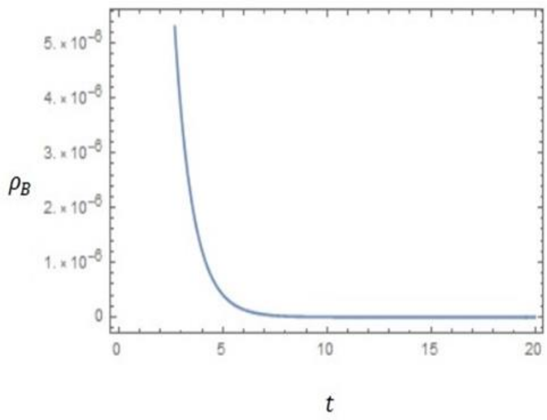

Fig.12: The figure shows the plot of ρ_B verses cosmic time t with $R_0 = 1, h_1 = 0.1, h_2 = 0.25, h_3 = 1.75, c_2 = 0.001, c_1 = 0.12$.

5. Conclusions

The present paper deals with the study of LRS Bianchi type I universe with Anisotropic Modified Holographic Ricci Dark Energy (MHRDE) viscous cosmological model in presence of magnetic field. The exact solutions of the model are obtained by using: (i) MHRDE density proposed by Chen and Jing [13], (ii) the bulk viscosity ζ is assumed to be of the form $\zeta = \zeta_0 + \zeta_1 H + \zeta_2 H^2$, where ζ_0, ζ_1 and ζ_2 are constants and $H$is the Hubble parameter and (iii) Hybrid Expansion Law (HEL). The bulk viscosity which plays an important role in the early evolution of the universe ultimately decreases with time. For $\zeta_0 = \zeta_1 = \zeta_2 = 0$ in the expression of the bulk viscosity, we get perfect fluid models. We have seen that the magnetic field is effective at the early stages of the universe. Highly ionized matter coupled with fields form neutral matter which causes the expansion of the universe during the evolution of the universe. It is observed from the analysis of the figures presented in this paper that the anisotropy of the universe and the skewness parameter approaches to zero at later age of the universe. Thus, the universe becomes isotropy at later times. The EOS parameter approaches to -1 at late times and ultimately it behaves like a cosmological constant. Thus, the model satisfies all the physical and geometrical observations which are in good agreement with the present-day observations.

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