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Linear and Nonlinear properties of Electron Acoustic Waves in a Viscous Plasma

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In this paper, the propagation of electron acoustic waves in a collision less Fermi plasma is investigated analytically by employing Quantum Hydrodynamic (QHD) model. The plasma system taken into account consists of hot and cold electrons and ions. The modified Korteweg-de Vries Burgers equation is derived in order to study the shock profile of electron acoustic waves numerically in a viscous plasma at the critical regime. The standard reductive perturbation method is employed to derive the mK-dV Burgers equation. Standard soliton solutions are used for quantitative and qualitative study of spatial and temporal evaluation of shock waves.

1. Introduction

Plasma is known as "The fourth state of matter", which has gained enormous interest for the last few years due to its existence in the earth (like Auroras, Lightning, Earth's Ionosphere, Fluorescent light, etc.) as well as in the universe (like Solar Wind, Van Allen Belts and Nebula, etc.). In a plasma system, there are roughly equal number of positively and negatively charged particles which make it electrically conducting medium. When the atoms in a gas, become ionized it produces plasma, which exhibits collective behaviour. In some cases, plasma also consists of neutral and dust particles, etc.

Recently there has been a great deal of interest in the study of different aspects of quantum plasma because of its large range of applications in metal nanostructures[1], ultra-small electronic devices[2], laser-produced plasmas[3] as well as in astrophysical plasmas[4] such as white dwarfs, neutron stars and pulsars, etc. when the species of plasma, like electrons and ions are at high densities, low temperature, and the inner particle distance is smaller compared to de-Broglie wavelength then the plasma considered to be degenerate quantum plasma. Much of the study of the properties of degenerate ionized matter in quantum plasma is inspired by the pioneering works of Bohm, Pines, and Levine. The popular Schr¨odinger–Poisson formulations describe the dynamics of plasma at quantum scale called Quantum Hydrodynamic (QHD)[5]. QHD has been used to study the collective process in quantum degenerate gasses in plasma. The QHD Model essentially is the extension of the usual classical fluid model in plasma, where a quantum correction term, 'Bohm potentials' [6], [7], [8] appears in the equation of motion of charged particles. Many researchers have studied the different electrostatics plasma modes in quantum plasma using QHD models [5]. The study of electron acoustic wave (EA) has received noticeable attention due to its existence in Laboratory experiments [9] as well as in space [10],[11],[12]. EA wave was first shown by Watnabe and Taniuti[13]. Strongly excited EA waves can readily form nonlinear structures like solitons, shocks, double layers, etc. This wave exists in two temperature (hot and cold) electrons and stationary ion plasma where the cold electrons provide the inertia and the hot electrons provide the necessary restoring force [14]. The ions play the role of neutralizing background [15-17]. The frequency of these wave is much larger than ion frequency so that EAW are basically high-frequency dispersive plasma. The EAWs are shown to give rise to stable oscillations under the conditions $\frac{T_{eh}}{T_{ec}} \geq 10$ (with T_{eh} is the temperatures of hot electrons and T_{ec} is the temperature of cold electrons) and the number density of cold electron is much smaller than hot electron as the oscillation time scale of EAW is much larger than the oscillation time scale of hot electron and $0.2 \leq \frac{n_{ec}}{n_{eh}} \leq 0.8$. Otherwise, it will be heavily damped.

Plasma contains a wide variety of waves as it behaves like a fluid and the long-range interaction between the particles in it. Those waves are solitary waves, shock waves, double layers, etc. Here we discuss shock wave, which is characterized by an abrupt change in pressure, temperature, and density of the medium. Shock structures have been experimentally studied by Anderson et al [18]. These waves are formed due to a delicate balance between nonlinearity and dissipation. Several different mechanisms, like landau damping, viscosity, wave-particle interaction are responsible for the formation of shock wave. When a medium has dissipation and dispersion then the dynamics of the nonlinear wave can be adequately described by K-dVB equation. The Burger term is responsible for the generation of shock waves due to the viscosity term. At the critical regime, the dynamics of the nonlinear wave are governed by modified K-dVB equation. In this article, we study the dispersion relation of electron acoustic wave and propagation of shock wave in the critical regime of collisionless plasma with the help of modified K-DVB equation.

2. Basic formulation

Consider three-component unmagnetized, quantum dense plasma consisting of non- relativistic inertial cold, and inertialess hot electron fluids and static positive ions. Thus, at equilibrium, we have $z_i n_{i0} = n_{c0} + n_{h0}$, where n_{c0}, n_{h0} , n_{i0} are the equilibrium number density of cold electrons, hot electrons, and positive ions respectively. A one-dimensional QHD model is described by continuity equation, momentum equation and closed by the Poisson's equation. In the momentum equation the quantum statistical pressure is given by

$$P_j = \frac{m_j v_{Fj}^2 n_j^3}{3n_{j0}^2} \tag{1}$$

Where, j = h for hot electron, m_j is the mass, $V_{Fj} = \sqrt{\frac{2k_B T_{Fj}}{m_j}}$ is the Fermi thermal speed, T_{Fj} is the Fermi temperature, and k_B is the Boltzmann constant. n_j is the number density with the equilibrium value n_{j0} and Bohm potential is the quantum diffraction term. The governing equation of nonlinear dynamics of electron acoustic wave in a degenerate quantum plasma system, may be written as:

$$\frac{\partial n_h}{\partial t} + \frac{\partial}{\partial x}(n_h u_h) = 0 \tag{2}$$

$$\frac{\partial n_c}{\partial t} + \frac{\partial}{\partial x}(n_c u_c) = 0 \tag{3}$$

$$0 = \frac{1}{m_e}\left[e\frac{\partial \varphi}{\partial x} - \frac{1}{n_h}\frac{\partial p_h}{\partial x} + \frac{\hbar^2}{2m_e}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_h}}\frac{\partial^2 \sqrt{n_h}}{\partial x^2} \right) \right] \tag{4}$$

$$\left(\frac{\partial}{\partial t} + u_c \frac{\partial}{\partial x} \right) u_c = \frac{1}{m_e}\left[e\frac{\partial \varphi}{\partial x} + \frac{\hbar^2}{2m_e}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_c}}\frac{\partial^2 \sqrt{n_c}}{\partial x^2} \right) \right] + \eta_c \frac{\partial^2 u_c}{\partial x^2} \tag{5}$$

$$\frac{\partial^2 \varphi}{\partial x^2} = 4\pi e(n_c + n_h - z_i n_i) \tag{6}$$

Where the subscript c, h, and i are used to denote cold, hot electrons, and ions respectively. u and p are described the fluid velocity and degenerate pressure, $\hbar$ is the Planck's constant divided by 2π, φ is the electrostatic wave potential and $z_i e$ is the charge of the ion.

We now introduced the following normalization:

$$x \to x\omega_c/v_{Fh}, t \to t\,\omega_c, \varphi \to e\varphi/2K_B T_{Feh}, n_j \to n_j/n_{j0},$$
$$n_i \to n_i/n_{i0}, u_j \to u_j/v_{Fh}, \eta_c \to \eta_c\,\omega_c/v_{Fh}^2$$

Where, $(\omega_c) = \sqrt{\frac{4\pi n_{c0} e^2}{m_e}}$ is the cold electron plasma frequency.

The Normalized Equations are,

$$\frac{\partial n_h}{\partial t} + \frac{\partial}{\partial x}(n_h u_h) = 0 \tag{7}$$

$$\frac{\partial n_c}{\partial t} + \frac{\partial}{\partial x}(n_c u_c) = 0 \tag{8}$$

$$0 = \left[\frac{\partial \varphi}{\partial x} - n_h\frac{\partial n_h}{\partial x} + \frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_h}}\frac{\partial^2 \sqrt{n_h}}{\partial x^2} \right) \right] \tag{9}$$

$$\left(\frac{\partial}{\partial t}+u_c\frac{\partial}{\partial x}\right)u_c=\left[\frac{\partial\varphi}{\partial x}+\frac{H^2}{2}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_c}}\frac{\partial^2\sqrt{n_c}}{\partial x^2}\right)\right]+\eta_c\frac{\partial^2 u_c}{\partial x^2}$$

(10)

$$\frac{\partial^2\varphi}{\partial x^2}=\left(n_c+\frac{n_h}{\delta}-n_i\frac{\delta_1}{\delta}\right)$$

(11)

Where, $H=\frac{\hbar\omega_c}{2k_BT_{Feh}}$ is the dimensionless quantum diffraction parameter. δ is the ratio between equilibrium cold electron density to equilibrium hot electron density $\left(\delta=\frac{n_{c0}}{n_{h0}}\right)$ and $\delta_1$is the ratio between equilibrium ion density to equilibrium hot electron density $\left(\delta_1=\frac{z_in_{i0}}{n_{h0}}\right)$. The charge neutrality at equilibrium reads $\delta=\delta_1-1$.

3. Analytical study

a. *Derivation of linear dispersion relation*

In order to study the nonlinear behaviour of electron acoustic wave in two temperature electron ion plasma, we make the following perturbation expansion for the field quantities n_h, u_h, n_c, u_c and φ about their equilibrium values,

$$\begin{bmatrix}n_h\\u_h\\n_c\\u_c\\\varphi\end{bmatrix}=\begin{bmatrix}1\\u_0\\1\\u_0\\\varphi_0\end{bmatrix}+\varepsilon\begin{bmatrix}n_h^{(1)}\\u_h^{(1)}\\n_c^{(1)}\\u_c^{(1)}\\\varphi^{(1)}\end{bmatrix}+\varepsilon^2\begin{bmatrix}n_h^{(2)}\\u_h^{(2)}\\n_c^{(2)}\\u_c^{(2)}\\\varphi^{(2)}\end{bmatrix}+\cdots \quad (12)$$

We assume that all field variables varying as $\exp\left[i\left(kx-\omega t\right)\right]$.By direct substitution method we get the real dispersion relation:

$$\left\{\frac{\omega_r^2+u_0^2\left(k_r^2-k_i^2\right)-2u_0\omega_rk_r}{k_r^2-k_i^2}\right\}+\eta u_0k_i=\frac{H^2}{4}\left(k_r^2-k_i^2\right)+\frac{\delta\left\{1+\frac{H^2}{4}\left(k_r^2-k_i^2\right)\right\}}{1+\delta\left[\left(k_r^2-k_i^2\right)+\frac{H^2}{4}\left\{\left(k_r^2-k_i^2\right)-4k_r^2k_i^2\right\}\right]} \quad (13)$$

b. *Derivation of the modified K-dV Burgers equation*

Here we derived Modified Korteweg-de Vries Burgers equation for multicomponent plasma containing warm and cold distributed electron and neutralizing ionic background. To derive the equation of motion for the nonlinear electron acoustic wave, the perturbation technique is employed. And stretched variables are defined as-

$$\xi=\varepsilon\left(x-v_0t\right),\qquad \tau=\varepsilon^3t,$$
$$\eta=\varepsilon\eta_0$$

Where, ε is a dimensionless smallness parameter which characterizes the strength of non-linearity and weakness of dispersion. V_0 is the phase velocity of the wave.

Modified Kd-VB equation obtained which is given by-

$$\frac{\partial\varphi^{(1)}}{\partial\tau}+p\varphi^{(1)^2}\frac{\partial\varphi^{(1)}}{\partial\xi}+q\frac{\partial^2\varphi^{(1)}}{\partial\xi^2}-r\frac{\partial^3\varphi^{(1)}}{\partial\xi^3}=0$$

$$\frac{\partial\varphi}{\partial\tau}+p\varphi^2\frac{\partial\varphi}{\partial\xi}+q\frac{\partial^2\varphi}{\partial\xi^2}-r\frac{\partial^3\varphi}{\partial\xi^3}=0$$

(14)

Here, we put $\varphi^{(1)}=\varphi$ and

$$p=\frac{\frac{15}{2\left(v_0-u_0\right)^6}-\frac{3}{2\delta}}{\frac{2}{\left(v_0-u_0\right)^3}},q=-\frac{\eta_0}{2},r=\frac{\frac{H^2}{4\left(v_0-u_0\right)^4}+\frac{H^2}{4\delta}-1}{\frac{2}{\left(v_0-u_0\right)^3}}$$

It is clearly seen that if viscous coefficient $\eta_0=0$, then equation (14) reduces to modified K-dV equation with $q = 0$.The cause of dissipation is the viscous coefficient. By using the transformation relation $\chi = \xi - M\tau$ (where M is Mach number i.e., speed of the frame) we get the traveling wave solution of modified K-dV Burger equation. Here χ is the new phase variable i.e., combining both ξ and τ as a single variable. We obtain the following soliton solutions:

$$\varphi_1 = \pm\sqrt{\frac{6r}{p}}\left[1+\coth\left(\xi-8rt\right)\right]$$

$$\varphi_2 = \pm\sqrt{\frac{6r}{p}}\left[1+\tanh\left(\xi-8rt\right)\right] \tag{15}$$

$$\varphi_3 = \pm 3\sqrt{\frac{6r}{p}}\left[2+\tanh\left(\xi-32rt\right)+\coth\left(\xi-32rt\right)\right]$$

4. Result and discussion

a. Real dispersion relation

Here, we consider only the real segment of dispersion relation. The imaginary component implies collisionless damping which violates the energy conservation principle. The quantum diffraction term (H), the streaming velocity (u_0), the viscosity coefficient (η) and the ratio of equilibrium number density of cold electrons, and hot electrons (δ) are the parameters.

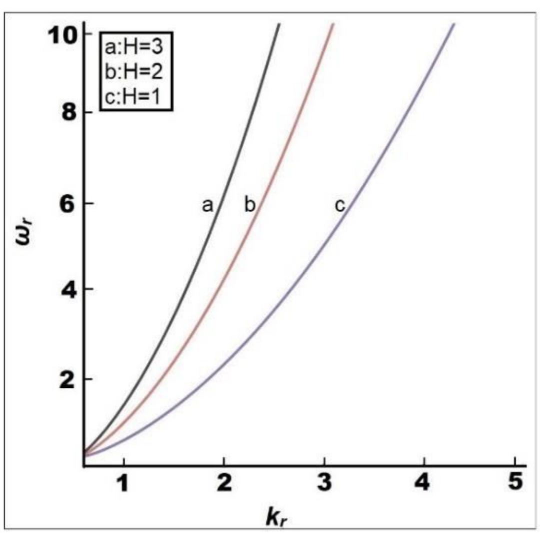

Fig. 1: Real dispersion for different quantum diffraction parameter (H)

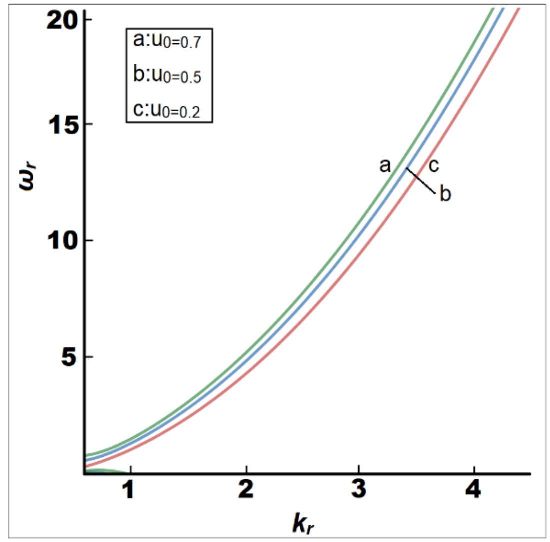

Fig. 2: Real dispersion for different streaming velocity (u_0)

Since the frequency (ω_r), and wavenumber (k_r) can't be negative that's why we take only first quadrant while plotting ω_r vs k_r (irrespective of parameters). In that particular case, we fix the value of k_i at 0.5. From the expression of real dispersion relation, we find that there arises a singularity at k_i=k_r=0.5. That's why we confined the range of $k_r \geq 0.6$. Here the phase velocity is insignificant. In fig. 1, we have plotted the real dispersion relation (ω_r vs k_r) by varying quantum diffraction term (H) and considering other parameters as constant. For increasing the value of H, the curves become steeper. In fig. 2, we have plotted the real dispersion relation by varying the streaming velocity (u_0) and considering other parameters as constants. Here also the curves become steeper by increasing the value of streaming velocity.

b. Shock profile

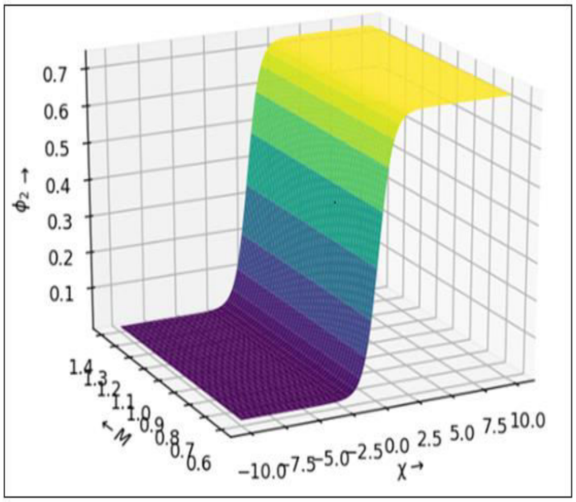

Fig. 3: Spatial extent of EA wave for different Mack number (M) considering δ=0.1, H=2, v_0=0.6, u_0=0.2

For electron acoustic wave Mach number (M) varies from 0.6 to 1.4. From fig. 3, we can show that with the increment of the value of new phase variable (χ) the shock (an abrupt change in φ_2) appears. But the nature of the curve remains unchanged with the variation of Mach number (M).

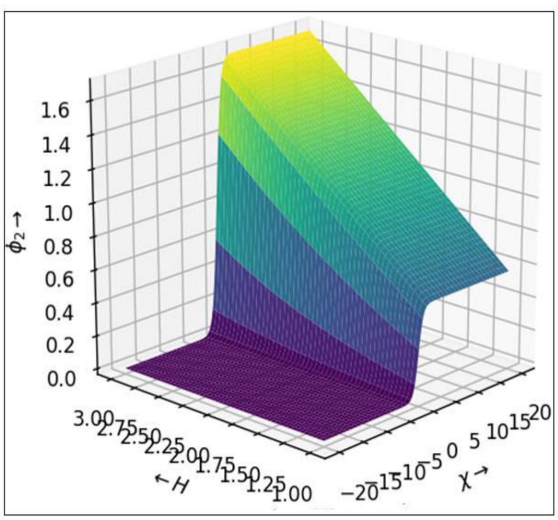

Fig. 4: Spatial extent of EA wave for different quantum diffraction parameter (H) considering δ=0.1, v_0=0.4, u_0=0.2

From fig. 4, we can show that with the increment of quantum diffraction parameter H (in quantum regime) the shock becomes stronger.

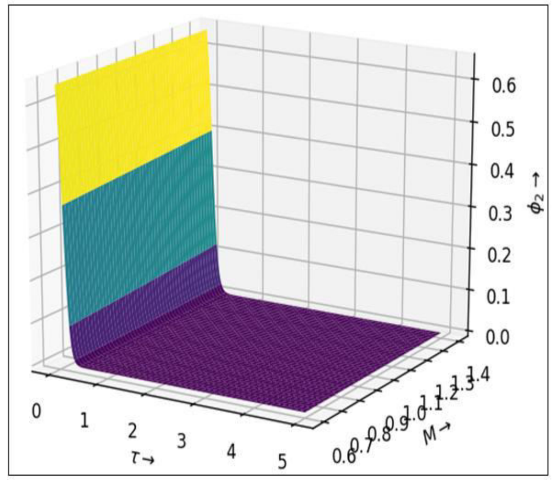

Fig. 5: Temporal evaluation of the electrostatic potential for different Mack number (M) considering δ=0.1, H=2, v_0=0.6, u_0=0.2

In fig.5, we show that there is a sudden full of electrostatic potential at $\tau = 0$ .But with the increment of time and Mach number (M) the electrostatic potential does not change i.e. stationary.

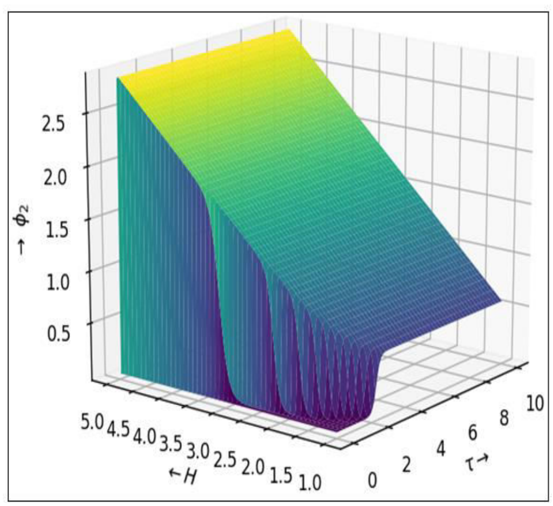

Fig. 6: Temporal evaluation of the electrostatic potential for different value of quantum diffraction parameter (H) considering δ=0.1, v_0=0.4, u_0=0.2

In fig.6, we show the temporal evaluation of the shock from semi-classical regime to quantum regime. For the lower value of H more time required to get the shock. With the increment of H, we get strong shock within a short time.

5. Conclusions

In this paper, we have investigated the propagation of electron acoustic waves in a collisionless unmagnetized Fermi plasma and have derived the modified K-dV Burgers equation to study the shock profile. We have plotted real dispersion relation for different quantum diffraction parameters and different streaming velocities and have shown that higher values of both parameters make the curves sharper. We have also plotted the spatial and temporal extent of the shock with different parameters namely Mach number and quantum diffraction parameter. Here also the quantum diffraction parameter is responsible for a rapid and strong shock in case of spatial extent.

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