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Study of Small Amplitude Ion-Acoustic Bunched Solitary Waves in a Plasma with Streaming Ions and Thermal Electrons

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The basic set of equations describing linear and nonlinear properties of ion-acoustic solitary waves in a two-component plasma consisting of Maxwell electrons and comparatively cold ions which has got a streaming motion, in the limit of long wavelengths, is being considered here. First, a dispersion relation is obtained and then modified Korteweg-de Vries (mKdV) equation is derived by means of a reductive perturbation approach based on the smallness of the wavenumber. The dependence of small amplitude bunched solitary structures on various plasma parameters such as ion streaming velocity (u_j), ratio between electron and ion temperature (σ_e), ratio between electron and ion masses (μ) and mach number (M) have been studied and corresponding conclusions have been drawn.

1 Introduction

In plasma physics, the theory of one-dimensional ion-acoustic waves is a standard topic in the domain of nonlinear waves. In its simplest form, one deals with a cold collision less plasma in which the dynamical behavior of the ions is determined by the presence of electrons. As a result, ion-acoustic waves show up.Due to the dynamical balance that is established between effects of nonlinearity and dispersion, solitons arise. Solitons are defined as a spatially localized, pulse shaped stable nonlinear construct which retains their shape, identity and energy in mutual collision and, therefore, the exact solution to a large number of nonlinear partial differential equation. There are several equations that exhibit soliton solutions, e.g., Korteweg-de Vries (KdV) [1-3], Washimi and Taniuti [4] were the first to show that propagation of ion-acoustic solitary waves in a plasma can be represented by the KdV equation, even though several authors have derived and solved KdV and mKdV equations for different homogenous and inhomogeneous plasma models since then. modified KdV(mKdV) [5] , nonlinear Schrodinger equations and double layers [7,8]. From the basic plasma physics [9] and other related literature we come to know that when the pumping power exceeds a certain value for passively mode-locked fiber lasers, more than one pulse will emerge in an anomalously dispersive cavity because of quantization of solitary energy. These multiple-solitons will form various dynamic patterns due to the interaction among solitons for dispersive and continuous waves. This is called soliton bunching where many identical soliton pulses group themselves in a tight packet. We find that there exist anisomerous dispersive waves in the bunching spectrum when we compare the generic soliton spectrum of the nonlinear Schrodinger equation (NLSE). Thus we come to the conclusion that the soliton interactions mediated through the radiative dispersive waves play a more important role in the formation of soliton bunching.

2 Basic formulation

2.1 Governing equations

Here considering an electron-ion plasma consisting of Boltzmann electrons and assuming that the plasma particles behave as a one-dimensional Fermi gas at 0K and therefore the pressure law is given by

$$P = \frac{m_j V_{Fj} n_j^3}{3 n_{jo}^2} \tag{1}$$

Where j=e for electron and j=i for ions; m_j is the mass of the corresponding quantities;

$$v_{Fe} = \sqrt{\frac{2K_B T_{Fe}}{m_e}} \tag{2}$$

= Fermi speed of electron, T_{Fe} = Fermi temperature and K_B=Boltzmann Constant; n_j = number density with the equilibrium value n_{j0}. Considering the set of QHD equations in the unnormalized form,

$$\frac{\partial n_e}{\partial t} + \frac{\partial n_e u_e}{\partial x} = 0 \tag{3}$$

$$\frac{\partial n_i}{\partial t}+\frac{\partial n_i u_i}{\partial x}=0 \quad (4)$$

$$\left(\frac{\partial}{\partial t}+u_i\frac{\partial}{\partial x}\right)u_i=\frac{1}{m_i}\left[Q_i\frac{\partial \phi}{\partial x}+\mathrm{n}_i\frac{\partial^2 u_i}{\partial x^2}\right] \quad (5)$$

$$0=\frac{1}{m_e}\left[Q_e\frac{\partial \phi}{\partial x}-\frac{1}{n_e}\frac{\partial P_e}{\partial x}+\frac{\hbar^2}{2m_e}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}\right]\right] \quad (6)$$

$$\frac{\partial^2\phi}{\partial x^2}=4\pi(Q_e n_e-Q_i n_i) \quad (7)$$

Where, u_i =Velocity of ion, Q_i =Charge of ion, η_i =viscosity coefficient, m_e =mass of electron, Q_e =charge of electron, P_e =Pressure of electron, ϕ=electrostatic potential Considering the Boltzmann distribution functions for electrons,

$$n_j=n_{j0}\exp\left(-\frac{e\phi}{K_B T}\right)$$

Investigating the propagation of solitary waves in a unmagnetized homogeneous plasma, the governing equations for IAM are as follows:

$$\frac{\partial n_i}{\partial t}+\frac{\partial n_i u_i}{\partial x}=0 \quad (8)$$

$$\left(\frac{\partial}{\partial t}+u_i\frac{\partial}{\partial x}\right)u_i=\frac{1}{m_i}\left[Q_i\frac{\partial \phi}{\partial x}+\mathrm{n}_i\frac{\partial^2 u_i}{\partial x^2}-\frac{1}{n_i}\frac{\partial P_i}{\partial x}\right] \quad (9)$$

$$\frac{\partial^2\phi}{\partial x^2}=4\pi(Q_e n_e-Q_i n_i) \quad (10)$$

Following Normalization scheme is considered:

$$\bar{x}\to\left(x\omega_{pe}/v_{Fe}\right)$$

$$\bar{t}=t\omega_{pe}$$

$$\bar{\phi}=\frac{e\phi}{2K_B T}$$

$$\bar{n}_j=\frac{n_j}{n_0}$$

$$\bar{u}_j=\frac{u_j}{V_{Fe}}$$

Where $\omega_{pe}=\sqrt{\frac{4\pi n_{e0}\epsilon^2}{c^2}}$ is the electron plasma oscillation frequency and $V_{Fe}=\sqrt{\frac{2K_B T_{Fe}}{m_e}}$ is the Fermi thermal speed of electrons. Using above normalization scheme, following normalized equations are obtained:

$$\frac{\partial n_i}{\partial t}+\frac{\partial n_i u_i}{\partial x}=0 \quad (11)$$

$$\left(\frac{\partial}{\partial t}+u_i\frac{\partial}{\partial x}\right)u_i=\mu\frac{\partial \phi}{\partial x}+\eta_i\frac{\partial^2 u_i}{\partial x^2}-n_i\frac{\partial n_i}{\partial x} \quad (12)$$

$$\frac{\partial^2\phi}{\partial x^2}=\left(\left(1+\sigma_e\phi+\sigma_e^2\frac{\phi^2}{2!}+\cdots\right)-Z_i n_i\right) \quad (13)$$

where

$$\mu=\frac{m_e}{m_i};\sigma_e=-2\frac{T_{Fe}}{T_i}$$

2.2 Linear dispersion relation

To investigate the linear and non-linear behaviour of IAW from the basic equation, the field variables in terms of smallness parameter âˆˆ about their equilibrium values are expanded. Now the perturbation expansion of these field variables is given by,

$$\begin{bmatrix}\mathrm{n_j}\\ \mathrm{u_j}\\ \phi\end{bmatrix}=\begin{bmatrix}1\\ \mathrm{u_0}\\ \phi_0\end{bmatrix}+\epsilon\begin{bmatrix}\mathrm{n_j^{(1)}}\\ \mathrm{u_j^{(1)}}\\ \phi^{(1)}\end{bmatrix}+\epsilon^2\begin{bmatrix}\mathrm{n_j^{(2)}}\\ \mathrm{u_j^{(2)}}\\ \phi^{(2)}\end{bmatrix}+\cdots \quad (14)$$

Assuming that all the field variables are varying as exp[i(kx-wt)] so that

$$n_i^{(1)}=n_i^{(1)}\exp(i(kx-\omega t))$$

$$u_i^{(1)}=u_i^{(1)}\exp(i(kx-\omega t))$$

$$\phi^{(1)}=\phi^{(1)}\exp(i(kx-\omega t))$$

Here k = wave number and ω = wave frequency, the dispersion relation for IAW is obtained as:

$$\omega^2+\mathrm{k}^2\mathrm{u_i^{(0)}}^2+2\omega\mathrm{k}\mathrm{u_i^{(0)}}+\mathrm{i}\left(\omega-\mathrm{k}\mathrm{u_i^{(0)}}\right)\mathrm{n_i}\mathrm{k}^2=\frac{\mathrm{Z_i^2}\mu_\mathrm{i}\mathrm{n_i^{(0)}}}{\left(\left(\frac{\sigma_e}{\mathrm{k}^2}\right)-1\right)} \quad (15)$$

where the pressure term in the momentum equation is neglected. Considering $u_i^{(0)}=0$ and ni (0)=1 we get,

$$\omega^2+\mathrm{i}n_i\omega k^2=\frac{Z_i^2\mu_i}{\frac{\sigma_e}{k^2}-1} \quad (16)$$

Breaking k and ω into real and imaginary parts and equating the corresponding parts, two equations, one for real part and other for imaginary part are obtained: Taking $\omega_i=0$ and k_i as parameter we get,

$$\omega_r=\eta_i\mathrm{k}_r\mathrm{k}_i+\sqrt{\mathrm{n_i^2}\mathrm{k_r^2}\mathrm{k_i^2}-\frac{\mathrm{Z_i^2}\mu_\mathrm{i}\left\{\sigma_\mathrm{e}\left(\mathrm{k_r^2}-\mathrm{k_i^2}\right)-\left(\mathrm{k_r^2}+\mathrm{k_i^2}\right)^2\right\}}{\left(\sigma_\mathrm{e}-\mathrm{k_r^2}+\mathrm{k_i^2}\right)^2+4\mathrm{k_r^2}\mathrm{k_i^2}}} \quad (17)$$

This is the relation which represents Landau damping of the ion-acoustic wave as given in Fig. (2).

This is the required dispersion relation as given in Fig. (1). Taking $\omega_r=0$ and k_r as parameter we get,

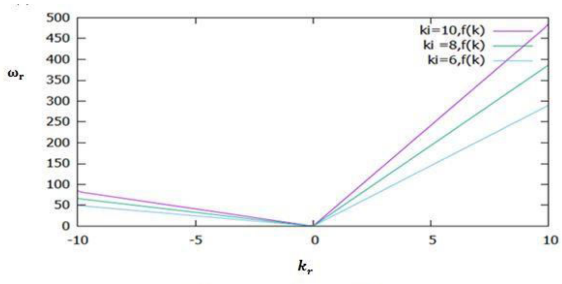

Fig 1: Dispersion Characteristics for different k_i

From Fig.1, the dispersion curves become steeper as k_i increases whereas the dispersion curves are almost independent of other plasma parameters because the electrons provide neutralizing effect. As for the damping curves as given by Fig.2, they represent a somewhat parabolic nature with decreasing steepness with the increase in the value of k_r. Thus, it can be said that smaller is the wavelength of the ion-acoustic wave (higher k_r), sharper will be the damping as the scattering of the wave would be more. This is because from Rayleigh scattering, it is known that scattering is inversely proportional to the fourth power of wavelength.

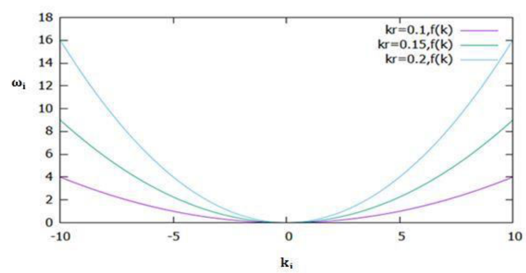

Fig 2: Damping effects for different k_r

2.3 *Derivation of mKdv equation*

The mKdV model describes nonlinear wave propagation of small amplitude IAW in plasma. The equation describing the electron-ion plasma waves is derived using the standard reductive perturbation technique. To derive the mKdV equations, introducing the stretched coordinates:

$$\xi = \epsilon(x - v_0 t), \tau = \epsilon^3 t, \eta = \epsilon \mathrm{n}_0 \qquad (18)$$

where v_0 = wave speed, ϵ = smallness parameter which characterizes the strength of nonlinearity, η = small variations in perpendicular direction. Now,

$$\frac{\partial}{\partial x} = \epsilon \frac{\partial}{\partial \xi}, \frac{\partial}{\partial t} = -v_0 \epsilon \frac{\partial}{\partial \xi} + \epsilon^3 \frac{\partial}{\partial \tau}.$$

Substituting in the normalized equations neglecting the viscosity term and equating the coefficient of like terms we obtain:

$$u_i^{(0)} = R_1 \mathrm{n}_i^{(1)}; n_i^{(2)} = R_2 \mathrm{u}_i^{(2)} + (n_i^{(1)})^2;$$

$$u_i^{(1)} = R_4 \phi^{(1)}; n_i^{(1)} = \frac{R_4}{R_1} \phi^{(1)};$$

$$(n_i^{(1)})^2 = R_5 n_i^{(2)}; u_i^{(2)} = R_6 (n_i^{(1)})^2$$

Where, $R_1 = (v_0 - u_i^{(0)}); R_2 = \frac{1}{R_1}; R_4 = [\frac{\mu_i R_i}{(1-R_1^2)}];$

$$\mathrm{R}_5 = \frac{2\left(\frac{-\mu_i}{\sigma_e} - 1 + \mathrm{R}_1^2\right)}{3R_1^2 + 1}; R_6 = \frac{1}{R_5 R_2} - R_1$$

The obtained mkdV equation is:

$$\frac{\partial \phi}{\partial \tau} + ab\phi^2 \frac{\partial \phi}{\partial \xi} + b \frac{\partial^3 \phi}{\partial \xi^3} = 0 \qquad (19)$$

where we have replaced $\phi^{(1)}$ by ϕ for the sake of simplicity. Equation (19) is known as the mKdV equation. Where $\phi^2 \frac{\partial \phi}{\partial \xi}$ the nonlinear term which describes the steepening of the wave and $b \frac{\partial^3 \phi}{\partial \xi^3}$ is the dispersive term which describes the spreading or broadening of the wave. The solitary wave structure is a balance between non-linear and dispersive effects. Here

$$ab = \frac{3R_4^4 R_6}{R_1^2} + \frac{3R_4^2}{R_5 R_1^3}; b = \frac{\mu}{\sigma_e R_4}$$

The solution of the mKdV equation can be obtained by introducing

$$\zeta = \xi - M\tau$$

and the solution obtained is

$$\psi = \psi_m sech\,[\tfrac{\zeta}{\Delta}] \qquad (20)$$

Where $\psi_m = \sqrt{\frac{6M}{ab}}$, $\Delta = \sqrt{\frac{b}{M}}$; Here ψm is the amplitude of the wave and Δ is the width, M is the Mach number.

We see that only those values of R_1 are valid for which the wave amplitude does not turn out to be complex. This is because in a plasma, the propagation of a real wave cannot be represented by a wave having a complex amplitude. As it turns the wave amplitude is real only when $-1 < R_1 < 1$. However, in this range of R_1, the width of the wave always comes out complex. Hence instead of a solitary wave we get bunched solitary structures. In stationary structures, the energy of the wave is localized, however in bunched solitons, the energy is distributed in the plasma by the different solitons. The amplitude of the waves are varying because even though the total energy of the waves are conserved, the energy is being reorganized or redistributed among the components. Whereas in stationary solitons we concern ourselves with dissipation loss, in bunched solitons the energy is carried by the different modes. Hence the correlation is a not so prominent. We get a discontinuity in the wave at u_i= 0.45 and 2.4(Fig.7 and FIg.8.) due to the complex nature of the width of the wave in the range $-1 < R_1 < 1$. Other than these discontinuities the wave shows almost continuous pattern over a long range of u_i.

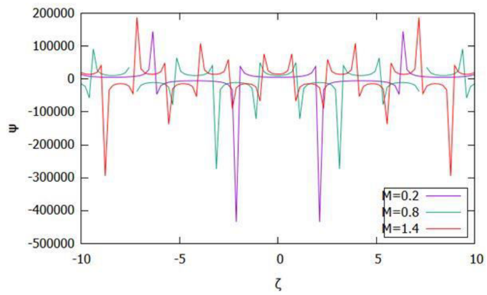

Fig 3: Solitary profiles for different Mach number (M)

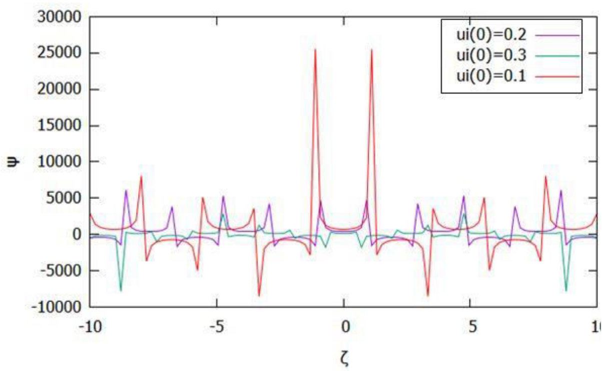

Fig 4: Solitary profiles for different streaming parameters (u_{i0})

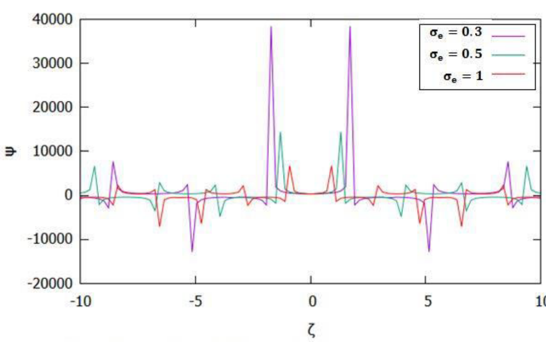

Fig 5: Solitary profiles for different electron-to-ion temperature ratio (σ_e)

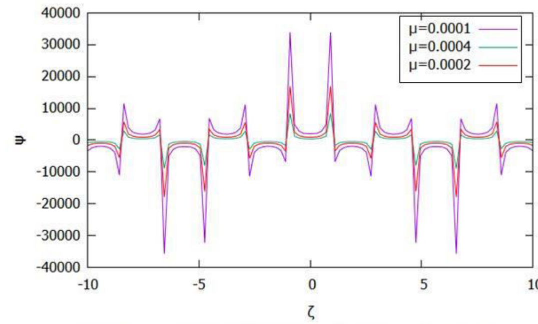

Fig 6: Solitary profiles for different electron-to-ion mass ratio (μ)

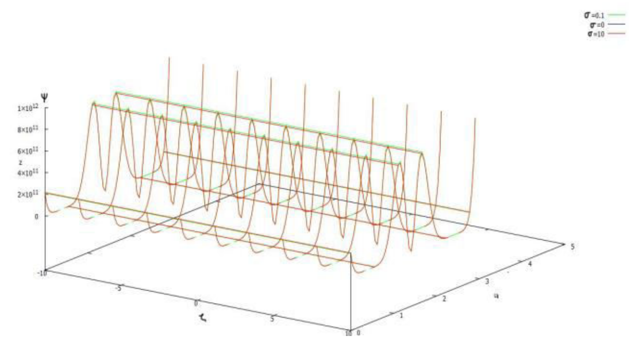

Fig 7: Discontinuities in the stationary wave pattern

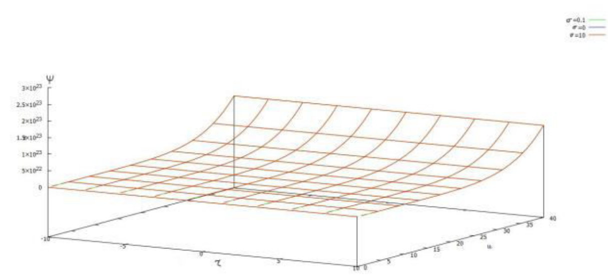

Fig 8: Continuum in the progressive wave

3 Results and conclusion

In this paper, the linear and non-linear characteristics of a two-component plasma consisting of ions and Boltzmann electrons have been studied.

The dispersion characteristics have been found to depend on k_i mainly and are almost independent of the plasma parameters. Similarly, the damping characteristics have been found to depend mainly on k_r. The mKdV equation that describes the small amplitude bunched solitary waves have been derived and its dependence on plasma parameters such as streaming velocity ($u_i^{(0)}$), Mach number (M), electron to ion temperature ratio (σ_e), electron to ion mass ratio (μ) have been evaluated.We have discussed the reason for occurrence of such bunched solitary waves and also what the variation of their amplitude and their propagation signify.The work may be further extended with variable charges on the ions, or by taking a three-component plasma consisting of non-thermal electrons and Boltzmann positrons. This system can also be studied by incorporating dust particles.

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