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Two dimensional ion-acoustic solitons in electron beam plasma

Manveet Kaur[1], Rupinder Kaur[2], N. S. Saini[3]

Physics Department, Guru Nanak Dev University, Amritsar-143005, Punjab, India.

E-mail: nssaini.phy@gndu.ac.in

Stationary nonlinear localized electrostatic (ES) waves may be excited when an electron beam is injected into a plasma. In the present investigation, the propagation properties of two dimensional ion-acoustic solitons have been studied in a plasma composed of ion fluid, hot electrons obeying Cairns distribution and embedded with electron beam. Electron beams exist in different space and astrophysical environments and influence the properties of nonlinear structures. In this paper, we consider model equations and derive the KP equation using reductive perturbation technique. Using its solution, numerical analysis is carried out. It is seen that negative potential ion-acoustic solitons are observed under the influence of variation of beam density, beam velocity and non-thermal parameter. The findings of this investigation may be of great importance to understand the nonlinear phenomena in the upper layer of the magnetosphere where Cairns distributed hot electrons, ions and electron beam may exist.

1. Introduction

Nonlinear waves have great importance and vital role in understanding physics of various nonlinear phenomena in space [1-3] and astrophysical environments [4-5]. In Earth's magneto-tail region the formation of solitary potential structures (both compressive and rarefactive) have been admitted by various satellite observations. The presence of electron beam is revealed by various satellites [6-10]. Also, it was frequently observed that electron beam coexists with ion acoustic waves in region of space and due to the presence of electron beam the characteristics of ion acoustic waves [11- 12] also get modified. Abundance of observations affirm the presence of non-thermal charged particles in various types of environments [13-16]. In magnetosphere and solar wind [17-18], the presence of energetic charged particles with non-thermal tails is revealed by various satellite observations. Cairns et al., [19] established Cairns distribution for these types of charged particles which exhibit non-thermal tail. The observations of Freja [20] and Viking [21] satellites substantiate that properties of ion acoustic waves are reformed due to presence of charged particles following non-thermal distribution. Numerous investigations have been reported with Cairns distribution [19], [22-24]. Recently, Singh and Saini [24] observed breather structures and peregrine solitons in a polarized space dusty plasma using Cairns distribution. The nonlinear ion acoustic waves [25-27] and electron acoustic waves [27-28] with electron beam have been introduced. Lakhina et al., [27] studied the large amplitude ion-acoustic and electron acoustic solitary waves in an unmagnetized multi component plasma comprising of cold background electrons and ions, a hot electron beam and a hot ion beam using Sagdeev pseudopotential technique.

Five decades ago, Boris Kadomtsev and Vladimir Petviashvili [29] introduced a nonlinear equation in two dimensional system with lump solution which elaborate the progression of long nonlinear waves of small amplitude which are slow dependent on transverse coordinates. An innumerable exploration has been done to study the transverse perturbation by deriving the Kadomtsev-Petviashvili (KP) equation [29] and its solution [30-33] in various types of plasmas. The study of Solitary waves of the Kadomstev-Petviashvili equation in warm dusty plasma with variable dust charge, two temperature ion and non-thermal electron was presented by H. R. Pakzad [30]. Masood and Rizvi [31] investigate the two-dimensional propagation of nonlinear ion acoustic shock and solitary waves in an unmagnetized plasma consisting of non-thermal electrons, Boltzmannian positrons, and singly charged hot ions streaming with relativistic velocities and observed that the wave dispersion increases with increase in the non-thermal electron population and results into loss of the strength of the ion acoustic shock wave and leads to an increase in amplitude of soliton due to the absence of dissipation in the KP equation. Saini et al.,[32] investigated both analytically and numerically the characteristics of dust acoustic (DA) solitary waves in a dusty plasma system comprising of dust as a fluid and superthermal electrons as well as ions. To our best knowledge, two-dimensional ion acoustic soliton with electron beam in Cairns distributed plasma has not been reported yet. In present study, we have tried to explore the two dimensional soliton in a plasma comprising of ion fluid, hot electrons obeying Cairns distribution and immersed with electron beam. We have used reductive perturbation technique to derive KP equation and study its solution. The investigation is arranged as follows: in Sect. II, the basic set of governing equations is presented. In Sect. III, numerical analysis has been illustrated. In Sect. IV, the conclusions are bestowed.

2. Basic equations

We consider three component collisionless unmagnetized plasma comprising electron beam with charge q_e= -e and mass m_e, cold ions with charge q_i = Ze and mass m_i and non-thermal electrons. The electrons are Cairns distributed, electron beam and ions are modelled by set of fluid equations as follows:

$$\frac{\partial N_i}{\partial T}+\frac{\partial (N_i U_i)}{\partial X}+\frac{\partial (N_i V_i)}{\partial Y}=0, \quad (1)$$

$$\frac{\partial U_i}{\partial T}+U_i\frac{\partial U_i}{\partial X}+V_i\frac{\partial U_i}{\partial Y}=-\frac{q_i}{m_i}\frac{\partial \Phi}{\partial X}, \quad (2)$$

$$\frac{\partial V_i}{\partial T}+U_i\frac{\partial V_i}{\partial X}+V_i\frac{\partial V_i}{\partial Y}=-\frac{q_i}{m_i}\frac{\partial \Phi}{\partial Y}, \quad (3)$$

and for the electron beam,

$$\frac{\partial N_b}{\partial T}+\frac{\partial (N_b U_b)}{\partial X}+\frac{\partial (N_b V_b)}{\partial Y}=0, \quad (4)$$

$$\frac{\partial U_b}{\partial T}+U_b\frac{\partial U_b}{\partial X}+V_b\frac{\partial U_b}{\partial Y}=-\frac{e}{m_e}\frac{\partial \Phi}{\partial X}, \quad (5)$$

$$\frac{\partial V_b}{\partial T}+U_b\frac{\partial V_b}{\partial X}+V_b\frac{\partial V_b}{\partial Y}=-\frac{e}{m_e}\frac{\partial \Phi}{\partial Y}. \quad (6)$$

the Poission equation encloses the system

$$\frac{\partial^2 \Phi}{\partial X^2}+\frac{\partial^2 \Phi}{\partial Y^2}=-4\pi e\,(n_i Z-n_e-n_b). \quad (7)$$

where n_i, n_e, n_b are the number densities of ions, non-thermal electrons and beam electrons respectively and u_i, u_b are the mean velocity of ions and the velocity of beam. Φ denote electrostatic potential. At equilibrium the charge neutrality condition is $n_{i0}Z = n_{e0} + n_{b0}$, implicit $Z = a+\varpi$ where $a = \frac{n_{e0}}{n_{i0}}$ and $\varpi = \frac{n_{b0}}{n_{i0}}$. The velocity of electrons is very high so it cannot express by Maxwellian distribution because the high energy electrons can only be expressed by Cairns distribution [19], so the number density of electrons in normalized form is given as:

$$n_e = n_{e0}\,[1+H_1\phi+H_2\phi^2+\cdots], \quad (8)$$

Where, $H_1 = 1-\Lambda$ and $H_1 = 1/2$, where $\Lambda = \frac{4\alpha_1}{(1+3\alpha_1)}$ and α_1 is non-thermality parameter that depicts non-thermal effect in phase space of Cairns distribution. To make the system dimensionless, we normalize the equations from Eq. (1-7) and get two-fluid model equations as:

The ion equations are:

$$\frac{\partial n_i}{\partial t}+\frac{\partial (n_i u_i)}{\partial x}+\frac{\partial (n_i v_i)}{\partial y}=0, \quad (9)$$

$$\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x}+v_i\frac{\partial u_i}{\partial y}=-\frac{\partial \phi}{\partial x}, \quad (10)$$

$$\frac{\partial v_i}{\partial t}+u_i\frac{\partial v_i}{\partial x}+v_i\frac{\partial v_i}{\partial y}=-\frac{\partial \phi}{\partial y}. \quad (11)$$

Electron beam equations are:

$$\frac{\partial n_b}{\partial t}+\frac{\partial (n_b u_b)}{\partial x}+\frac{\partial (n_b v_b)}{\partial y}=0, \quad (12)$$

$$\frac{\partial u_b}{\partial t}+u_b\frac{\partial u_b}{\partial x}+v_b\frac{\partial u_b}{\partial y}=-\frac{1}{\mu_m Z}\frac{\partial \phi}{\partial x}, \quad (13)$$

$$\frac{\partial v_b}{\partial t}+u_b\frac{\partial v_b}{\partial x}+v_b\frac{\partial v_b}{\partial y}=-\frac{1}{\mu_m Z}\frac{\partial \Phi}{\partial y}, \quad (14)$$

The Poission's equation is:

$$\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}=-n_i+\frac{a}{z}(1+H_1\phi+H_2\phi^2)+\frac{n_b}{z}. \quad (15)$$

The number densities n_i, n_b are normalized by n_{i0} unperturbed ion density), the electric potential $\phi = \Phi/\Phi_0$ (where $\Phi_0 = K_B T_e/e$) and velocities $U_i = u_i/c_s, U_b = u_b/c_s$ (where c_s (sound speed) = $\left(\frac{ZK_B T_e}{m_i}\right)^{1/2}$). Time variable is scaled ion plasma frequency $\frac{1}{\omega_{p,i}} = \left(\frac{m_i}{4\pi n_{i0} Z^2 e^2}\right)^{1/2}$ and space variable is scaled $\lambda_{D,e} = \left(\frac{K_B T_e}{4\pi n_{i0} e^2}\right)^{1/2}$. The ratio of mass of electron to mass of ions is denoted by μ_m (i.e. $\mu_m = m_e/m_i$. In this paper we have followed reductive perturbation technique, by using stretching coordinates, the independent variables can be stretched as follows:

$$\xi = \epsilon(x - Vt),\ \eta = \epsilon^2 y \quad and \quad \tau = \epsilon^3 t, \tag{16}$$

V is the phase velocity of wave along x-direction. The dependent variables are expanded as:

$$f = f^{(0)} + \sum_{j=1}^{\infty} \epsilon^{2j} f^{(j)} \quad \text{and}$$
$$g = g^{(0)} + \sum_{j=1}^{\infty} \epsilon^{2j+1} g^{(j)}. \tag{17}$$

where $f = n_i, n_b, u_i, u_b, \phi$ and $g = v_i, v_b$ and values of n_i, n_b, u_i, u_b, ϕ at equilibrium are 1, ϖ, 0, u_0, 0 and values of v_i and v_b are zero at equilibrium. Now substituting the stretching coordinates Eq. (16) and perturbation Eq. (17) in equations from Eq. (9-15), from lowest-order of ϵ we get:

$$n_i^{(1)} = \frac{1}{V} u_i^{(1)}, \tag{18}$$

$$u_i^{(1)} = \frac{1}{V} \phi^{(1)}, \tag{19}$$

$$\frac{\partial v_i^{(1)}}{\partial \xi} = \frac{1}{V} \frac{\partial \phi^{(1)}}{\partial \eta}, \tag{20}$$

$$-n_i^{(1)} + \frac{a}{Z} H_1 \phi^{(1)} + \frac{1}{Z} n_b^{(1)} = 0 \tag{21}$$

$$n_b^{(1)} = \frac{-\varpi}{(-V+u_0)} u_b^{(1)}, \tag{22}$$

$$u_b^{(1)} = \frac{1}{\mu_m Z(-V+u_0)} \phi^{(1)}, \tag{23}$$

$$\frac{\partial v_b^{(1)}}{\partial \xi} = \frac{1}{\mu_m Z(-V+u_0)} \frac{\partial \phi^{(1)}}{\partial \eta}, \tag{24}$$

and dispersion relation is given by:

$$\frac{1}{V^2} + \frac{\varpi}{\mu_m Z^2 (u_0 - V)^2} - \frac{a}{Z} H_1 = 0. \tag{25}$$

next higher order in ϵ gives the second higher order equations:

$$\frac{\partial n_i^{(1)}}{\partial \tau} - V \frac{\partial n_i^{(2)}}{\partial \xi} + \frac{\partial n_i^{(1)} u_i^{(1)}}{\partial \xi} + \frac{\partial u_i^{(2)}}{\partial \xi} + \frac{\partial v_i^{(1)}}{\partial \eta} = 0, \tag{26}$$

$$\frac{\partial u_i^{(1)}}{\partial \tau} - V \frac{\partial u_i^{(2)}}{\partial \xi} + u_i^{(1)} \frac{\partial u_i^{(1)}}{\partial \xi} + \frac{\partial \phi^{(2)}}{\partial \xi} = 0, \tag{27}$$

$$\frac{\partial v_i^{(1)}}{\partial \tau} - V \frac{\partial v_i^{(2)}}{\partial \xi} + u_i^{(1)} \frac{\partial v_i^{(1)}}{\partial \xi} + \frac{\partial \phi^{(2)}}{\partial \eta} = 0, \tag{28}$$

$$\frac{\partial n_b^{(1)}}{\partial \tau} - V \frac{\partial n_b^{(2)}}{\partial \xi} + u_0 \frac{\partial n_b^{(2)}}{\partial \xi} + \frac{\partial n_b^{(1)} u_b^{(1)}}{\partial \xi} + \varpi \frac{\partial u_b^{(2)}}{\partial \xi} + \varpi \frac{\partial v_b^{(1)}}{\partial \eta} = 0, \tag{29}$$

$$\frac{\partial u_b^{(1)}}{\partial \tau} - V \frac{\partial u_b^{(2)}}{\partial \xi} + u_b^{(1)} \frac{\partial u_b^{(1)}}{\partial \xi} + u_0 \frac{\partial u_b^{(2)}}{\partial \xi} - \frac{1}{\mu_m Z} \frac{\partial \phi^{(2)}}{\partial \xi} = 0, \tag{30}$$

$$\frac{\partial v_b^{(1)}}{\partial \tau} - V \frac{\partial v_b^{(2)}}{\partial \xi} + u_b^{(1)} \frac{\partial v_b^{(1)}}{\partial \xi} + u_0 \frac{\partial v_b^{(2)}}{\partial \xi} - \frac{1}{\mu_m Z} \frac{\partial \phi^{(2)}}{\partial \eta} = 0, \tag{31}$$

$$\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = -n_i^{(2)} + C_1 \phi^{(2)} + C_2 \phi^{2(1)} + \frac{1}{Z} n_b^{(2)}, \tag{32}$$

Where, $C_1 = \frac{a}{Z} H_1$ and $C_2 = \frac{a}{Z} H_2$. Now by make use of first order results in second order in order to eliminating the later order terms we obtain nonlinear KP equation as

$$\frac{\partial}{\partial \xi}\left(\frac{\partial \phi}{\partial \tau} + A\phi \frac{\partial \phi}{\partial \xi} + B \frac{\partial^3 \phi}{\partial \xi^3}\right) + C\left(\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2}\right) = 0, \tag{33}$$

Where, $A = \frac{\mu_m^2 Z^3 (V-u_0)^4 (-3+2C_2 V^4) + 3\varpi V^4}{-V\mu_m (V-u_0)(2\mu_m Z^2 (V-u_0)^3 + 2\varpi V^3)}$, (34)

$$B = \frac{V^3 \mu_m Z^2 (V-u_0)^3}{2\mu_m Z^2 (V-u_0)^3 + 2\varpi V^3}, \tag{35}$$

$$C = \frac{V\mu_m Z^2 (V-u_0)^3 (2 - C_1 V^2)}{2\mu_m Z^2 (V-u_0)^3 + 2\varpi V^3}, \tag{36}$$

Where, *A* is nonlinear coefficient, *B* is dispersion coefficient and C represents higher order coefficient. For quickness, we have replaced $\phi^{(1)}$ by ϕ. By assuming the traveling wave transformation $\chi = \xi + \eta - v\tau$ to attain stationary solution of KP equation (33), the velocity of the co-moving frame with solitary wave is denoted by v. The KP Eq. (33) is transformed to differential equation by adopting this single transformation, then we evolve out the stationary solution, after integrating with appropriate boundary conditions (e.g., $\phi(\chi)$, $\phi'(\chi) \to 0$ as $|\chi| \to \infty$), is as follows [32]

$$\phi = \phi_m Sech^2\left(\frac{\chi}{w_1}\right), \quad (37)$$

Where, $\phi_m = \frac{3(v-C)}{A}$
(38)

is peak amplitude of solitons with width

$$w_1 = 2\sqrt{\frac{B}{v-C}}. \quad (39)$$

3. Numerical analysis

The nonlinear coefficient (A) has only negative values so only negative potential structures are formed. It depends upon various parameters viz., beam density (ϖ), non-thermal parameter (α_1) etc. The numerical data for present investigation ϖ = 0.1 - 0.4, u_0 = 1.25, μ_m= 1/1836, is used from Nejoh and Sanuki [25]. The effect of various plasma parameters on nonlinear coefficient and soliton profile has also been analysed.
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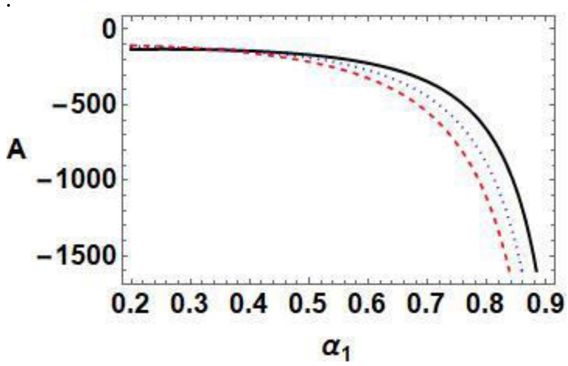
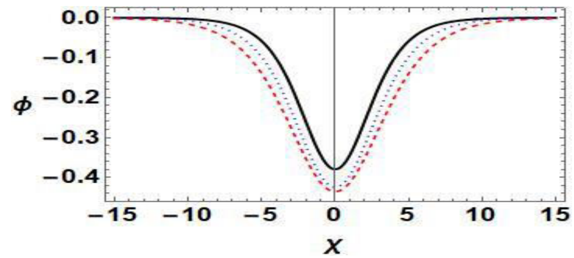
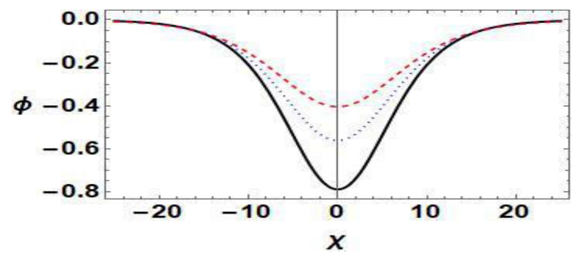


Fig. 1. Variation of nonlinear coefficient (A) with non-thermal parameter (α_1 for different values of beam density (ϖ), Black(Thick): ϖ = 0.15; Blue(Dotted): ϖ = 0.25; Red(Dashed): ϖ = 0.35, with u_0 = 1.25.

Fig.1 illustrates the effect of non-thermal parameter (α_1) on nonlinear coefficient (A) for different values of beam density (ϖ). It is observed that nonlinear coefficients A is always negative. It decreases with increase in density of beam (ϖ) and also increases with non-thermal parameter (α_1). The effect of non-thermal parameter (α_1) on profile of soliton with beam and without beam is demonstrated by Fig. 2(a) and Fig. 2(b) respectively. The amplitude of solitons is reduced in the presence of electron beam.



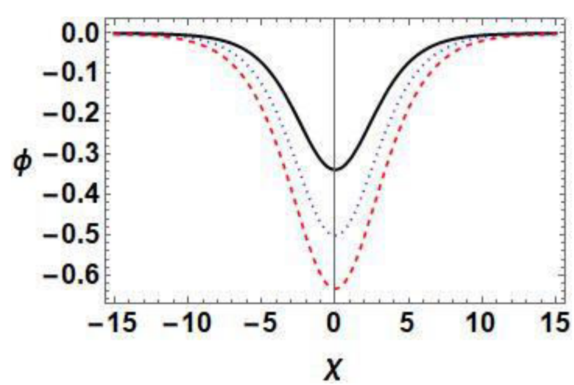
(a)



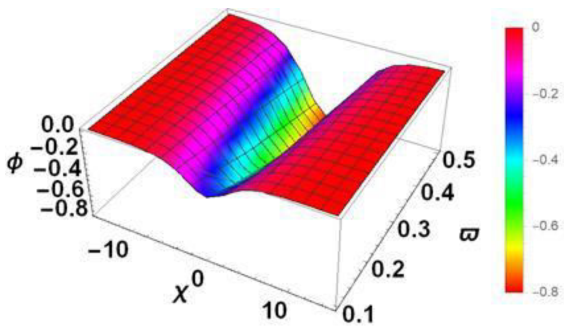
(b)

Fig.2 Variation of soliton profile for different values of non-thermal parameter (α_1) (a) with beam (ϖ = 0.2, u_0 = 1.25), Black(Thick): α_1 = 0.2; Blue(Dotted): α_1 = 0.3; Red(Dashed): α_1 = 0.4, (b) without beam (ϖ = 0, u_0 = 0), Black(Thick): α_1 = 0.20; Blue(Dotted): α_1 = 0.21; Red(Dashed): α_1 = 0.22.

Moreover with (without) electron beam the amplitude and width of soliton are increased (decreased) with increase in non-thermal parameter (α_1). The nonlinearity effect is sensitive to the variation in non-thermal parameter (α_1), and it dominates for large value of α_1 leading to increase in amplitude of ion-acoustic solitons.



(a)



(b)

Fig. 3. Variation of soliton profile for (a) different values of beam density (ϖ), Black(Thick): ϖ = 0.15; Blue(Dotted): ϖ = 0.25; Red(Dashed): ϖ = 0.35 with α_1 = 0.3 and u_0 = 1.25, (b) 3D plot.

Fig.3 (a) represents variation of soliton profile with electron beam density (ϖ). It is noticed that with increase in electron beam density (ϖ) the amplitude of negative potential soliton profile is enhanced along negative axis. Its three dimensional (3D plot) variation is given in Fig.3b.

4. Conclusions

In this investigation, we have presented the propagation properties of two dimensional ion-acoustic solitons in a three component unmagnetized plasma consisting of electron beam, cold ions and non-thermal electrons following Cairns distribution. Using reductive perturbation technique, The KP equation has been derived. Only compression type ion-acoustic solitons are observed. The role of electron beam and non-thermal parameter has been highlighted. The amplitude of soliton increases with increase in beam density (ϖ) and non-thermal parameter (α_1) and reduced in presence of electron beam. The role of beam parameter is very crucial to control the amplitude of solitons. With increase in beam density, nonlinearity is enhanced and leads to increase in amplitude along negative axis of soliton profile. The present study may shed light to understand nonlinear phenomena of upper layer of magnetosphere.

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