Compressive and Rarefactive Solitary Waves in Plasma with Cold Drifting Positive and Negative Ions

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Using the well known pseudo-potential method for the plasma (He^+,O^-), with the variation of different plasma parameter when Landau damping is neglected, we investigated both compressive and rarefactive solitary waves with first-order (ϕ_1) and second-order (ϕ_2) soliton profiles for the isothermal case, and compressive solitary waves with first (ϕ_1) and second (ϕ_2) order soliton profiles for non-isothermal case in fast mode for the cases of isothermal and non-isothermal collision-less single electron temperature plasma with cold positive and negative ions (including drifts), nature. These are shown in Figs. 1-4, respectively.

1. Introduction

The effect of higher order non-linearity on the propagation of non-linear ion acoustic waves in a collision-less plasma consisting of cold drifting positive and negative ions with either isothermal or non-isothermal electrons is very important in plasma physics. Compressive and rarefactive solitary waves in different types of plasmas have been extensively investigated by a large number of physicists [1-14] in a cold ion plasma with single or two electron species in modern plasma theory, and they studied in detail the conditions of compressive and rarefactive solitary waves for both small and large amplitude. In presence of negative ions, the rarefactive solitary waves were also investigated theoretically by many authors [15-17] incorporating different plasma parameters and experimentally by Cooney et al. [18]. The effect of negative ions on ion-acoustic solitary waves is more interesting than that of positive ions and two temperature electrons. So we take negative ions in addition to that of positive ions, and single temperature electron in the basic equation with special attention to velocities of both positive and negative ions, including their drifts. As a result of this, the rarefactive solitary waves are found for suitable negative ion concentration.

In the present paper, we investigate mainly the effect of negative ion concentration and drift velocities of both positive and negative ions on the formation of ion-acoustic solitary waves with cold positive and negative ions. In a three-component plasma consisting of electrons, positive and negative ions, Tagare et al. [19] investigated the effect of higher order non-linearity on the ion-acoustic solitary waves with cold ions in isothermal and in non-isothermal electrons by reductive perturbation method. We consider here the same problem in a different way with an emphasis on the drifts of both positive and negative ions by Sagdeev pseudo-potential method, which is more interesting than the previous one. Generally, two types of modes, namely a slow ion-acoustic mode and a fast ion-acoustic mode are found in an ion-acoustic solitary wave solution. The interesting case is that of the fast mode nature of solitons, which is observed in this problem and from which compressive and rarefactive solitary waves are seen due to the effect of negative ions concentration. Sec. 2 contains the required formulations where the exact pseudo-potential form and conditions for compressive and rarefactive solitary wave solutions are derived for single temperature electron plasma without using any approximations. The first-order (ϕ_1) and second-order (ϕ_2) solitary wave solutions for the compressive solitary waves are also discussed in this section. The phase velocity (V) for the bi-quadratic equation is discussed in this section. In Sec. 3, we have discussed the entire problem and concluding remarks are given in Sec. 4.

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2. Formulation

The normalized basic plasma fluid equations with cold positive ions, cold negative ions and hot isothermal electrons in an infinite one-dimensional collision-less un-magnetized plasma are given by

Continuity Equation

$$\frac{\partial N_\propto}{\partial t} + \frac{\partial}{\partial x}(N_\propto u_\propto) = 0 \quad (1)$$

Momentum Equation

$$\frac{\partial u_\propto}{\partial t} + u_\propto \frac{\partial u_\propto}{\partial x} = -\frac{Z_\propto}{Q_\propto}\frac{\partial \phi}{\partial x} \quad (2)$$

Poisson's Equation

$$\frac{\partial^2 \phi}{\partial x^2} = N_e - \sum_\propto Z_\propto N_\propto \quad (3)$$

Where, $Q_\propto$ (= m_n/m_p , m_n = mass of negative ion, m_p = mass of positive ion), $N_\propto$, $u_\propto$, $Z_\propto$, ϕ and N_e are the mass ratios of negative to positive ion masses, density, velocity of ions, charge of ions, electrostatic potential, and concentration of electrons. Moreover,

$$Z_\propto = 1,\ Q_\propto = 1 \text{ for } \propto = p \text{ (p = positive ion)}$$

$$Z_\propto = -Z,\ Q_\propto = Q \text{ for } \propto = n \text{ (n = negative ion)}$$

In the above equations from (1) to (3), we have normalized the velocities ($u_\propto$) by the characteristics velocity, $\sqrt{\frac{KT_e}{m}}$, all the densities ($N_\propto$) by the equilibrium value N_0 and the length by the electron Debye length of free electrons $\lambda_{De} = \sqrt{\frac{KT_e}{4\pi e^2 N_0}}$, whereas the electric potential ϕ is normalized by $\frac{KT_e}{e}$, so that the equations appear totally in dimensionless form. Here K is the Boltzmann constant, T_e denotes the constant temperature of the free electron and m is the mass of positive ion.

The boundary conditions are

$$u_\propto \to u_{\propto 0},\ N_\propto \to N_{\propto 0},\ \phi \to 0 \text{ as } |x| \to \infty \quad (4)$$

And the charge neutrality condition is

$$\sum_\propto Z_\propto N_{\propto 0} = 1 \quad (5)$$

For solitary wave solution, we use the transformation

$$\eta = x - V t \quad (6)$$

Where, V is the velocity of the solitary wave. From Eqns. (1), (2) and (3), we get finally after using (6) and (4)

$$N_\propto = \frac{N_{\propto 0}}{\sqrt{1 - \frac{2Z_\propto \phi}{Q_\propto (V - u_{\propto 0})^2}}} \quad (7)$$

Again, from Eqn. (7) the restriction on ϕ is

$$-\frac{Q}{2Z}(V - u_{n0})^2 < \phi < \frac{1}{2}(V - u_{po})^2 \quad (8)$$

Where, u_{p0} and u_{n0} are the drift velocities of positive (p) and negative (n) ions. Relation (8) is a very important inequality. The physical interpretation of inequality (8) is that the solitary waves are found only when the electric potential ϕ lies within the range of the inequality (8).

(a) Isothermal Case: In this case, the electron density is $N_e = e^{\phi}$ and then by using Eqns. (7) and (6), we get from Eqn. (3),

$$\frac{d^2\phi}{d\eta^2} = e^{\phi} - \sum_\propto \frac{Z_\propto N_{\propto 0}}{\sqrt{1 - \frac{2Z_\propto \phi}{Q_\propto (V - u_{\propto 0})^2}}} \quad (9)$$

Now Eqn. (9) can be written in the form

$$\frac{d^2\phi}{d\eta^2} = -\frac{\partial \psi}{\partial \phi} \quad (10)$$

Where

$$\psi(\phi) = 1 - e^{\phi} + N_{p0}(V - u_{po})^2\left[1 - \sqrt{1 - \frac{2\phi}{(V - u_{po})^2}}\right] + ZQN_{n0}(V - u_{n0})^2\left[1 - \sqrt{1 + \frac{2Z\phi}{Q(V - u_{n0})^2}}\right] \quad (11)$$

The function $\psi(\phi)$ is real for such values of ϕ, where ϕ satisfies the inequality (8) and beyond that value of ϕ, the Sagdeev pseudo-potential function $\psi(\phi)$ is complex. Moreover, the compressive and rarefactive solitary waves are found from the graphical representation of $\psi(\phi)$ for those values of ϕ which satisfy inequality (8) and the condition (16).

Now the condition for compressive solitary wave solution is

$$\psi'(\phi) > 0 \text{ at } \phi = \phi_{m_1},\ \phi_{m_1} > 0 \quad (12)$$

Where, ϕ_{m_1} is a value of ϕ lying in the range of the inequality (8).

By using Eqn. (11), relation (12) gives

$$N_{n0} > \frac{\left[e^{\phi_{m_1}} - \frac{1}{\sqrt{1-\frac{2\phi_{m_1}}{(V-u_{po})^2}}}\right]}{Z\left[\frac{1}{\sqrt{1-\frac{2\phi_{m_1}}{(V-u_{po})^2}}} - \frac{Z}{\sqrt{1+\frac{2Z\phi_{m_1}}{Q(V-u_{no})^2}}}\right]} \qquad (13)$$

Where, N_{p0} = 1 + Z N_{n0} (see Eqn. (5)).

Similarly, the condition for rarefactive solitary wave solution is

$$\psi^{/}(\phi) < 0 \quad \text{at } \phi = \phi_{m_2}\,,\ \phi_{m_2} < 0$$

Where, ϕ_{m_2} is that value of ϕ which satisfies inequality (8).

By using Eqn. (11), the above inequality gives

$$N_{n0} < \frac{\left[e^{\phi_{m_2}} - \frac{1}{\sqrt{1-\frac{2\phi_{m_2}}{(V-u_{po})^2}}}\right]}{Z\left[\frac{1}{\sqrt{1-\frac{2\phi_{m_2}}{(V-u_{po})^2}}} - \frac{Z}{\sqrt{1+\frac{2Z\phi_{m_2}}{Q(V-u_{no})^2}}}\right]} \qquad (14)$$

Where, N_{p0} = 1 + Z N_{n0} (see Eqn. (5)).

The condition for the existence of solitary wave solution is

$$\frac{\partial^2\psi}{\partial\phi^2} < 0 \quad \text{at } \phi = 0 \qquad (15)$$

The above inequality (15) gives,

$$\frac{Z^2N_{n0}}{Q(V-u_{no})^2} + \frac{N_{p0}}{(V-u_{po})^2} < 1 \qquad (16)$$

Now from Eqn. (10), expanding $\psi(\phi)$ in power series of ϕ and noting that

$$\psi(\phi) = 0 = \frac{\partial\psi}{\partial\phi} \text{ at } \phi = 0$$

we get

$$\frac{d^2\phi}{d\eta^2} = -\frac{\partial\psi}{\partial\phi} = C_1\phi + C_2\phi^2 + C_3\phi^3 + C_4\phi^4 + C_5\phi^5 + \qquad (17)$$

Where,

$$C_1 = \left[1 - \frac{N_{p0}}{(V-u_{p0})^2} - \frac{Z^2N_{n0}}{Q(V-u_{n0})^2}\right]$$

$$C_2 = \frac{1}{2}\left[1 - \frac{3N_{p0}}{(V-u_{p0})^4} + \frac{3Z^3N_{n0}}{Q^2(V-u_{n0})^4}\right]$$

$$C_3 = \frac{1}{6}\left[1 - \frac{15N_{p0}}{(V-u_{p0})^6} - \frac{15Z^4N_{n0}}{Q^3(V-u_{n0})^6}\right]$$

$$C_4 = \frac{1}{8}\left[\frac{1}{3} - \frac{35N_{p0}}{(V-u_{p0})^8} + \frac{35Z^5N_{n0}}{Q^4(V-u_{n0})^8}\right]$$

$$C_5 = \frac{1}{8}\left[\frac{1}{15} - \frac{63N_{p0}}{(V-u_{p0})^{10}} - \frac{63Z^6N_{n0}}{Q^5(V-u_{n0})^{10}}\right]$$

Taking terms up to ϕ^2 from Eqn. (17), the first order K-dV solitary wave solution [20] is obtained as

$$\phi_1 = \frac{3C_1}{2C_2} Sech^2\left(\sqrt{\frac{C_1}{4}}\,\eta\right) \qquad (18)$$

Again taking terms up to ϕ^3 from Eqn. (17), the higher order M-KdV solitary wave solution [20] is obtained as

$$\phi_2 = \frac{6C_1}{2C_2 + \sqrt{4C_2^2 - 18C_1C_3}\left[2Cosh^2\left(\sqrt{\frac{C_1}{4}}\eta\right) - 1\right]} \qquad (19)$$

The phase velocity (V) is obtained from Eqn. (16) by the equation

$$V^4 - 2(u_{p0} + u_{n0})V^3 + \left(u_{p0}^2 + u_{n0}^2 + 4u_{p0}u_{n0} - N_{p0} - \frac{Z^2N_{n0}}{Q}\right)V^2 + 2\left(\frac{Z^2N_{n0}u_{p0}}{Q} - u_{p0}u_{n0}^2 - u_{p0}^2u_{n0} + N_{p0}u_{n0}\right)V + \left(u_{p0}^2u_{n0}^2 - N_{p0}u_{n0}^2 - \frac{Z^2N_{n0}u_{p0}^2}{Q}\right) = 0 \qquad (20)$$

This is a bi-quadratic equation in V and gives four values (real or imaginary).

We are now focusing the effects of drift velocities of both ions, which are classified into four different cases:

Case 1. **Motion with unequal ion-drifts**

When the drifts of both positive and negative ions are present with unequal magnitudes i.e., $u_{p0} \neq u_{n0} \neq 0$, then we get from Eqn. (20)

$$V^4 - 2a_3V^3 + a_2V^2 + 2a_1V + a_0 = 0$$

Where,

$$a_3 = u_{p0} + u_{n0}$$

$$a_2 = u_{p0}^2 + u_{n0}^2 + 4u_{p0}u_{n0} - N_{p0} - \frac{Z^2N_{n0}}{Q}$$

$$a_1 = \frac{Z^2N_{n0}u_{p0}}{Q} - u_{p0}u_{n0}^2 - u_{p0}^2u_{n0} + N_{p0}u_{n0}$$

$$a_0 = u_{p0}^2u_{n0}^2 - N_{p0}u_{n0}^2 - \frac{Z^2N_{n0}u_{p0}^2}{Q}$$

The four values of V from the above bi-quadratic equation are obtained as follows:

$$V = \frac{1}{2}\left[(a_3 - \lambda_1) + \sqrt{(\lambda_1 - a_3)^2 - 4(\lambda_2 - \lambda_3)}\right]$$

$$V = \frac{1}{2}\left[(a_3 - \lambda_1) - \sqrt{(\lambda_1 - a_3)^2 - 4(\lambda_2 - \lambda_3)}\right]$$

$$V = \frac{1}{2}\left[(a_3 + \lambda_1) + \sqrt{(\lambda_1 + a_3)^2 - 4(\lambda_2 + \lambda_3)}\right]$$

$$V = \frac{1}{2}\left[(a_3 + \lambda_1) - \sqrt{(\lambda_1 + a_3)^2 - 4(\lambda_2 + \lambda_3)}\right]$$

Where,

$$\lambda_1^2 = a_3^2 - a_2 + 2\lambda_2$$

$$\lambda_1\lambda_3 = a_1 + \lambda_2a_3$$

$$\lambda_3^2 = \lambda_2^2 - a_0$$

$$\lambda_2 = \left[\frac{1}{2}\left(\lambda_4 + \sqrt{\lambda_4^2 - \lambda_5}\right)\right]^{\frac{1}{3}} + \left[\frac{1}{2}\left(\lambda_4 - \sqrt{\lambda_4^2 - \lambda_5}\right)\right]^{\frac{1}{3}} + \frac{1}{6}a_2$$

$$\lambda_4 = \frac{1}{6}a_2(a_0 + a_1a_3) + \frac{1}{108}a_2^3 - \frac{1}{2}(a_0a_2 - a_0a_3^2 - a_1^2)$$

$$\lambda_5 = \frac{4}{729}\left[3(a_0 + a_1a_3) + \frac{1}{4}a_2^2\right]^3$$

Case 2: Motion with equal ion-drifts

When the drifts of positive and negative ions are equal in magnitude i.e., $u_{p0} = u_{n0}$ = u (say) = a non zero number, then we get from Eqn. (20), the soliton velocity as

$$V = u \pm \sqrt{N_{p0} + \frac{Z^2N_{n0}}{Q}}$$

Or,

$$V = u + \sqrt{N_{p0} + \frac{Z^2N_{n0}}{Q}}$$

and

$$V = u - \sqrt{N_{p0} + \frac{Z^2N_{n0}}{Q}}$$

And the inequality, Eqn. (9), then reduces to the following form

$$-\frac{Q(V-u)^2}{2Z} < \phi < \frac{(V-u)^2}{2}$$

Case 3: Motion with only positive ion drift

When the drift of negative ion velocity is absent i.e., $u_{n0} = 0$ with $N_{n0} = 0$, then the soliton velocity with positive ion drift is obtained from Eqn. (16) as

$$V > u_{p0} \pm 1$$

i.e.,

$$V > u_{p0} + 1$$

and

$$V > u_{p0} - 1$$

This inequality is more general than Ref. [21]. In the absence of positive ion drift velocity i.e., u_{p0} = 0, the above inequality turns into V > 1 (taking positive sign only) and this supports Ref. [21]. The inequality given by Eqn. (9) then reduces to the form

$$0 < \phi < \frac{1}{2}(V - u_{p0})^2$$

Case 4: Motion without ion-drifts

When the drifts of positive and negative ions are absent i.e., $u_{p0} = 0$ and $u_{n0} = 0$, then we get from Eqn. (20), the soliton velocity $V = \sqrt{N_{p0} + \frac{Z^2N_{n0}}{Q}}$ (taking positive sign), which supports Ref. [15] and the inequality given by Eqn. (9) then reduces to the form

$$-\frac{QV^2}{2Z} < \phi < \frac{V^2}{2}$$

From the above four cases, we conclude that two types of modes of the phase velocities are obtained when drifts are considered. One type is known as fast mode, and the other type is slow mode. But two kinds of this mode of velocities are not found when drifts of both the ions are not considered i.e., motion without drift. This is the novelty of the drifts of both the ions. Two kinds of these modes of velocities depend on some parameters. The parameter determining the nature of soliton (i.e., compressive or rarefactive) is different for slow and fast modes. The fast mode nature is found from this problem and for that fast mode the parameter is the relative concentration of the two ion species. For the fast mode, it is found that there is a critical value of the negative ion concentration (N_{nc}) below which only compressive solitons exist and above which only rarefactive solitons exist.

From the inequality given by Eqn. (16), the critical negative ion concentration (N_{nc}) is as follows:

$$N_{nc} = \frac{Q(V-u_{n0})^2[(V-u_{p0})^2-1]}{Z[Z(V-u_{p0})^2+Q(V-u_{n0})^2]} \qquad (21)$$

Three cases may arise from Eqn. (21): (a) It is clear from (21) that for

$V - u_{n0} = 0$ or $V - u_{p0} = 1$, N_{nc} vanishes.

When $u_{p0} = 0 = u_{n0}$ then $N_{nc} = \frac{Q(V^2-1)}{Z(Z+Q)}$.

When $u_{p0} = u_{n0} = \mathrm{u}$ then $N_{nc} = \frac{Q[(V-u)^2-1]}{Z[Z+Q]}$

In the presence of negative ion and for isothermal single temperature electron plasma, the solitary waves are formed according to the following conditions:

$N_{n0} < N_{nc}$ (compressive solitary waves)
$N_{n0} = N_{nc}$ (compressive and rarefactive solitary waves)
$N_{n0} > N_{nc}$ (Rarefactive solitary waves)
Where, N_{n0} is the initial negative ion concentration and N_{nc} is the critical negative ion concentration.

(b) Non-isothermal case: In this case, the equations of cold positive and negative ions are given by Eqns. (1) to (3) as before. The expression for electron density (n_e) is given by Schamel (1972, 1973) as

$$N_e = 1 + \phi - \frac{4}{3} b_1 \phi^{\frac{3}{2}} + \frac{1}{2}\phi^2 - \frac{8}{15} b_2 \phi^{\frac{5}{2}} + \frac{1}{6}\phi^3 + \qquad (22)$$

Here the potential ϕ is normalized by (KT/e) and the electron density N_e by the unperturbed density N_0, where K is the Boltzman constant and b_1, b_2 and β are given below:

$$b_1 = \frac{1-\beta}{\sqrt{\pi}}, b_2 = \frac{1-\beta^2}{\sqrt{\pi}}, \beta = \frac{T_{ef}}{T_{et}} \qquad (23)$$

Here, T_{ef} is the constant temperature of the free electrons and T_{et} is that of trapped electrons. The term $\frac{4}{3} b_1 \phi^{\frac{3}{2}}$ introduces the contribution of resonant (both free and trapped) electrons to the electron density (N_e). When β = 1, i.e., when $b_1 = 0$ and $b_2 = 0$, the non-isothermal electron reduces to that of isothermal electron. The cases β = 1 and β = 0 correspond to the plasma having Maxwellian and flat topped distribution. It is obvious to derive the electron density for isothermal plasma by imposing $b_1 = 0$ and $b_2 = 0$, whereas for non-isothermal plasma, we have $0 < b_1 < \frac{1}{\sqrt{\pi}}$ and $0 < b_2 < \frac{1}{\sqrt{\pi}}$ (i.e., $b_1 \neq 0$, $b_2 \neq 0$).

Putting the value of N_e in Eqn. (3) and using similarly (4) and (5) with the well known form of Eqn. (10), the basic set of Eqns. (1) to (3) are reduced to

$$\frac{1}{2}\left(\frac{d\phi}{d\eta}\right)^2 + \psi(\phi, V) = 0 \qquad (24)$$

Where the Sagdeev potential is given by

$$\psi(\phi, V) = N_{p0}(V-u_{p0})^2\left[1-\sqrt{1-\frac{2\phi}{(V-u_{p0})^2}}\right] + Z\,Q\,N_{n0}(V-u_{n0})^2\left[1-\sqrt{1+\frac{2Z\phi}{Q(V-u_{n0})^2}}\right] - \phi - \frac{1}{2}\phi^2 - \frac{1}{6}\phi^3 + \frac{8}{15} b_1 \phi^{\frac{5}{2}} + \frac{16}{105} b_2 \phi^{\frac{7}{2}} \qquad (25)$$

For $b_1 = 0$ and $b_2 = 0$ (i.e., β = 1), Eqn. (25) reduces to the form of Eqn. (11) and in this case we will get the same inequality (Eqn. (16)) as a condition for the existence of solitary wave solution. Only compressive solitary wave solution is obtained in non-isothermal plasma and the condition given in Eqn. (12) is also satisfied in this case. Similarly by using Eqns. (22) and (7), we get from Eqn. (3)

$$\frac{d^2\phi}{d\eta^2} = A_1\phi - A_2\phi^{\frac{3}{2}} + A_3\phi^2 - A_4\phi^{\frac{5}{2}} + A_5\phi^3 + \ldots \qquad (26)$$

Where,

$$A_1 = \left[1 - \frac{N_{p0}}{(V-u_{p0})^2} - \frac{Z^2 N_{n0}}{Q(V-u_{n0})^2}\right]$$

$$A_2 = \frac{4}{3} b_1 = \frac{4}{3}\frac{1-\beta}{\sqrt{\pi}}$$

$$A_3 = \frac{1}{2}\left[1 - \frac{3N_{p0}}{(V-u_{p0})^4} - \frac{3Z^3N_{n0}}{Q^2(V-u_{n0})^4}\right]$$

$$A_4 = \frac{8}{15} b_2 = \frac{8}{15}\frac{1-\beta^2}{\sqrt{\pi}}$$

$$A_5 = \frac{1}{2}\left[\frac{1}{3} - \frac{5N_{p0}}{(V-u_{p0})^6} - \frac{5Z^4N_{n0}}{Q^3(V-u_{n0})^6}\right] \qquad (27)$$

Taking terms up to $\phi^{\frac{3}{2}}$ from Eqn. (26), we get

$$\frac{d^2\phi}{d\eta^2} = A_1\,\phi - A_2\,\phi^{\frac{3}{2}}$$

Thus the first order solitary wave solution (ϕ_1) in non-isothermal plasma is

$$\phi_1 = \left(\frac{5A_1}{4A_2}\right)^2 Sech^4\left(\sqrt{\frac{A_1}{16}}\,\eta\right)$$

Similarly, taking terms up to ϕ^2 from Eqn. (26), the second order solitary wave solution (ϕ_2) in non-isothermal plasma is obtained as

$$\phi_2 = \frac{1}{625\,A_3^4}[3A_2$$

$$-\sqrt{\frac{3}{2}(9A_2^2 - 25A_1A_3)}\;Sech\left(\frac{1}{2}\sqrt{A_1 - \frac{9A_2^2}{25A_3}}\,\eta\right)]^4$$

3. Discussion

By using the Sagdeev pseudo-potential approach, we observed that negative ions and drift velocities of ions affect the formation of ion-acoustic solitary waves in the plasma [22]. For the plasma (He^+,O^-) corresponding mass ratio Q = 4 and the restriction $N_{n0} < N_{nc}$, the ion-acoustic solitary waves under different plasma parameter variation are investigated. In all cases the fast mode nature with compressive and rarefactive solitary waves are seen. These results are shown in Figs. 1 and 2.

In Fig. 1, the Sagdeev pseudo-potential curves $[\psi(\phi) vs. \phi]$ of compressive solitary waves (ϕ >0) in the isothermal case are shown with the variation of positive and negative ion drift velocities [u_{p0},u_{n0}] and negative ion concentration (N_{n0}) for the plasma (He^+, O^-) having the constant mass ratio Q = 4 when V = 1.5 and Z = 1. Curves representing 'a_3' and 'a_4' show the variation of drift velocities [u_{p0} = 0.4, u_{n0} = 0.2; u_{p0} = 0.4, u_{n0} = 0.299] of the compressive solitary waves for isothermal plasma when V = 1.5, Q = 4, N_{n0} = 0.05 and Z = 1. It is evident from these figures that when drift velocities are increasing, the amplitudes are also increasing.

Again, the curves represented by 'a_1', 'a_2' and 'a_3' show the variation of negative ion concentration [N_{n0} = 0.05, 0.06, 0.07] of the compressive solitary waves for isothermal plasma when V = 1.5, u_{p0} = 0.4, u_{n0} = 0.2, Q = 4 and Z = 1. The amplitude in this case decreases when negative ion concentration N_{n0} increases, and the amplitude increases even in the absence of negative ion (N_{n0} = 0) for the same plasma.

Rarefactive solitary waves (ϕ < 0) are obtained in the presence of negative ion by the well known restrictions $N_{n0} > N_{nc}$ where N_{n0} is the initial negative ion concentration and N_{nc} is the critical negative ion concentration. For the plasma (He^+, O) corresponding to the mass ratio Q = 4, the rarefactive solitary waves are investigated in a collision-less single temperature electron plasma. These results are shown in Fig. 2.

Fig. 2 shows the profiles of the Sagdeev pseudo-potential curves $[\psi(\phi) vs. \phi]$ of rarefactive solitary waves (ϕ < 0) in isothermal case with the variation of negative ion concentration (N_{n0}) and drift velocities of positive and negative ions (u_{p0}, u_{n0}) for (He^+, O^-) plasma having mass ratio Q = 4 with V = 1.5 and Z = 1. Curves representing 'c_1', 'c_2' & 'c_3' show the variation of negative ion concentration [N_{n0} = 0.25, 0.30, 0.35] of the rarefactive solitary waves for isothermal plasma when V = 1.5, u_{p0} = 0.4, u_{n0} = 0.2, Q = 4 and Z = 1. From these figures, it is clear that as negative ion concentration N_{n0} increases (N_{n0} = 0.25, 0.30, 0.35), the amplitude of the rarefactive solitary waves (ϕ < 0) decreases, which is the same as in the case for compressive solitary waves (ϕ >0). Also the curves represented by 'c_3' & 'c_4' show the variation of drift velocities of positive and negative ions [u_{p0} = 0.4, u_{n0} = 0.2; u_{p0} = 0.3, u_{n0} = 0.21] of the rarefactive solitary waves for isothermal plasma when V = 1.5, Q = 4, N_{n0} = 0.25 and Z = 1. When the drift velocities of both positive and negative ions are increasing [u_{p0} = 0.5, u_{n0} = 0.3 and u_{p0} = 0.48, u_{n0} = 0.29] with V = 1.5, the amplitudes are also increasing as in the case of compressive solitary waves.

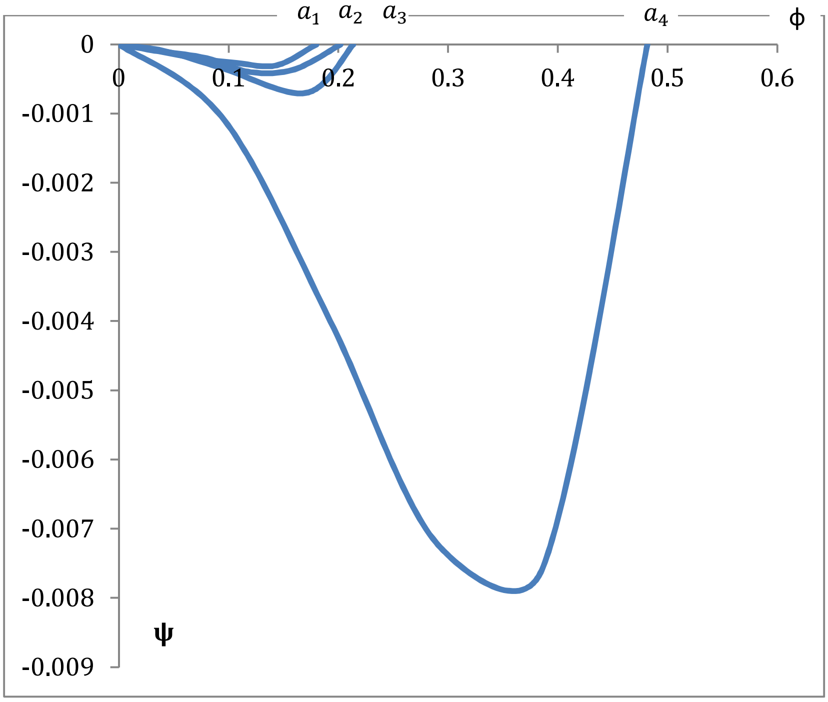
a_1 a_2 a_3 a_4 ϕ
0
0 0.1 0.2 0.3 0.4 0.5 0.6
-0.001
-0.002
-0.003
-0.004
-0.005
-0.006
-0.007
-0.008
ψ
-0.009

Fig.1

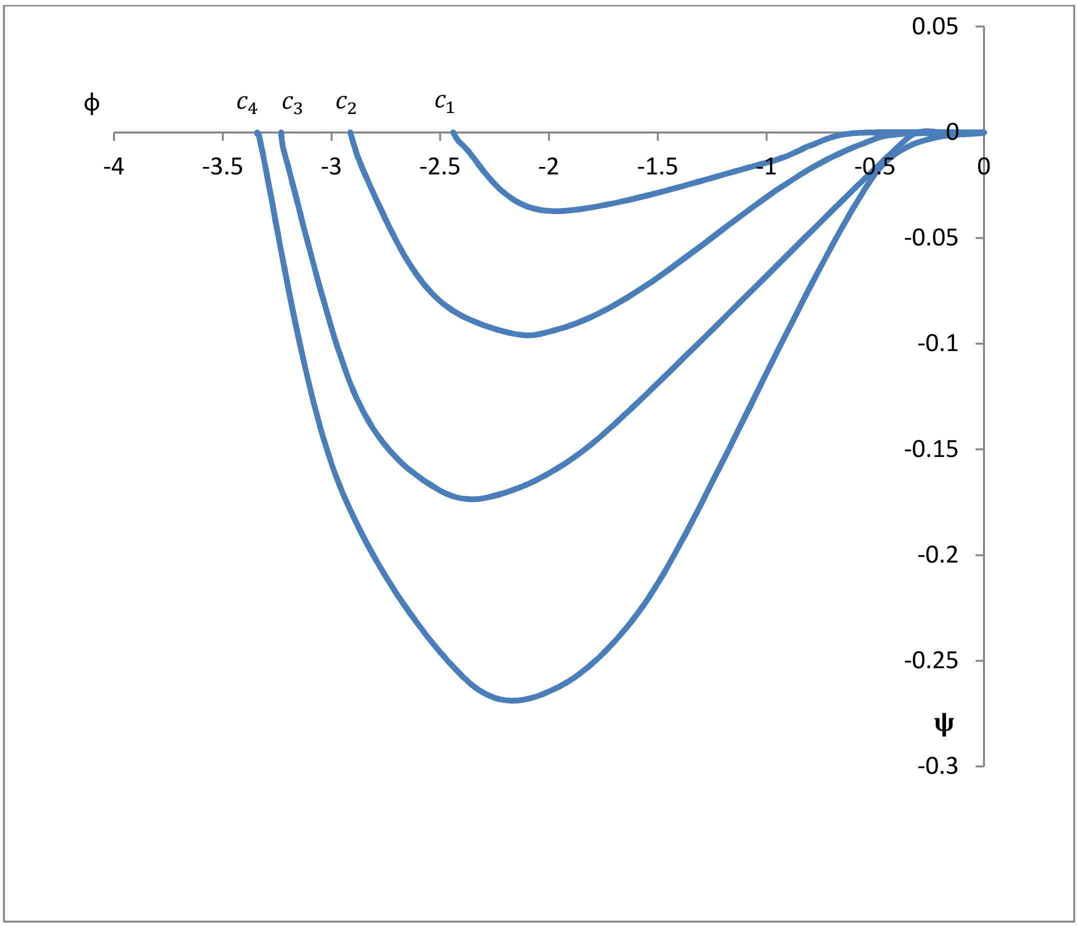
0.05
ϕ c_4 c_3 c_2 c_1
0
-4 -3.5 -3 -2.5 -2 -1.5 -1 -0.5 0
-0.05
-0.1
-0.15
-0.2
-0.25
ψ
-0.3

Fig.2

We now discuss the nature of the Sagdeev pseudo-potential curves for non-isothermal plasma from Eqn. (25). In this case, we only investigate the compressive solitary waves and no rarefactive solitary waves. These results are shown in Fig. 3.

In Fig. 3 the profiles of Sagdeev pseudo-potential curves $[\psi(\phi) vs. \phi]$ of compressive solitary waves (ϕ >0) for non-isothermal plasma are shown with the variation of drift velocities of positive (u_{p0}) and negative (u_{n0}) ions, negative ion concentration (N_{n0}) and the ratios of constant temperature of free electrons to the constant temperature of trapped electrons (β) for the plasma (He^+, O^-) with Z = 1, V = 1.5 with the constant mass ratio Q = 4. From the profiles of Sagdeev's pseudo-potential curve represented by 'a' and 'c', it is evident that when the drift velocities of positive ions (u_{p0}) are increasing, the amplitudes are decreasing, while the drift velocities of negative ions (u_{n0}) remain unchanged. This contradicts the isothermal case. Also the curves represented by 'b', 'c' and 'd' give us the important result, which states that the amplitude in this case are decreasing when the negative ion concentration (N_{n0}) is increasing. But in absence of negative ion (N_{n0} = 0), the amplitude will be maximum. Curves represented by 'c', 'e' and 'f' show that when β is increasing, the amplitudes are also increasing for the above plasma in non-isothermal case. But for β = 1 the non-isothermal situation turns into isothermal case and the profiles of $\psi(\phi)$ in this case coincide with Fig. 1, which is an interesting phenomena.

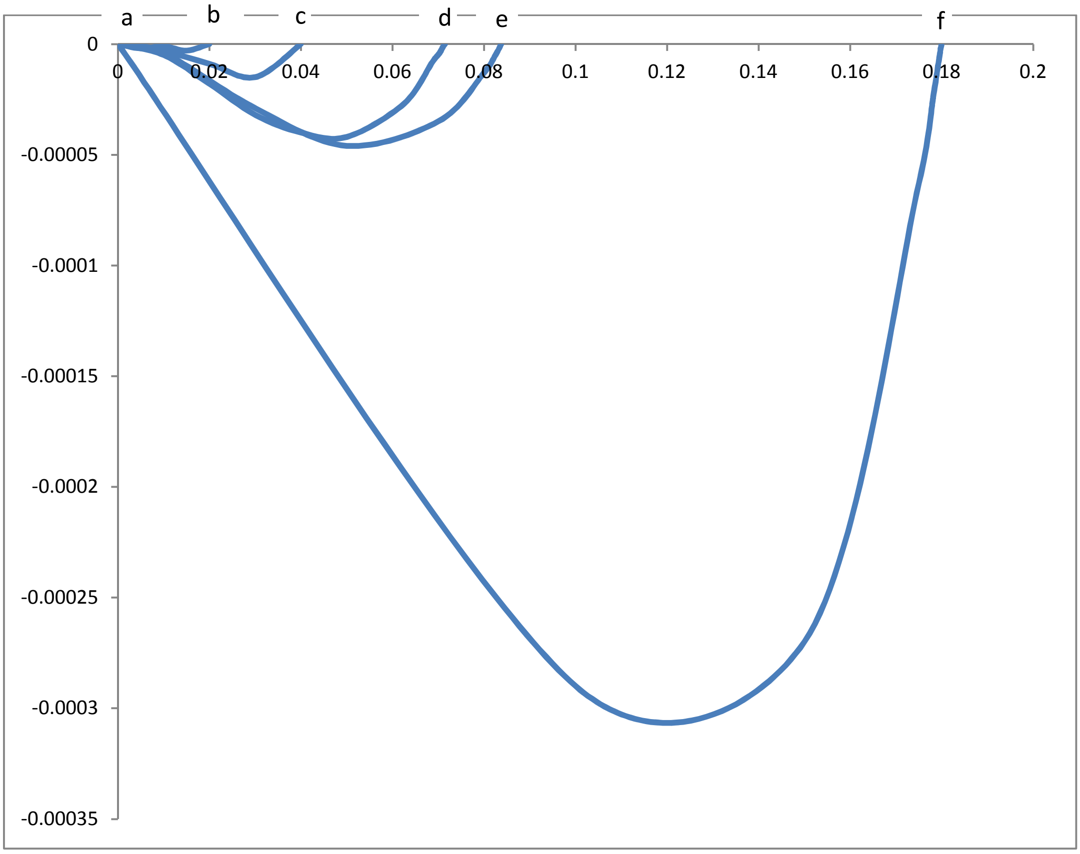

Fig.3

Now, the effect of first-order (ϕ_1) and second-order (ϕ_2) solitary wave solution of ion-acoustic solitary waves in a collision-less plasma consisting of positive and negative ions, and single temperature electron plasma are shown in Fig. 4 with the variation of different plasma parameters.

Fig. 4 shows the profiles of first-order (ϕ_1) and second-order (ϕ_2) solitary wave solutions $[\phi_1\ vs. \eta$ and $\phi_2 vs.\ \eta\ ]$ of non-isothermal plasma for compressive solitary waves (ϕ >0) with the variation of drift velocities of positive $[u_{p0}]$ and negative $[u_{n0}]$ ions as well as the variation of negative ion concentration (N_{n0}) and the variation of the ratios of constant temperature of free electrons to constant temperature of trapped electrons (β) for the plasma (He^+,O^-) with V = 1.5, Q = 4 , and Z = 1. In Fig. 4(a), curves represented by h_7, h_8, h_6, h_5, show the variation of drift velocities of positive and negative ions [u_{p0} = 0.4, u_{n0} = 0.2; u_{p0} = 0.3, u_{n0} = 0.21] of the first and the second order compressive solitary waves for non-isothermal plasma when V = 1.5, Q = 4, N_{n0} = 0.05, β =0. 1 and Z = 1. It is found from this figure that the second order soliton solution (ϕ_2) has W-type shape, but ϕ_1 and ϕ_2 are everywhere positive. The values of ϕ_1 are increasing for u_{p0} = 0.3, u_{n0} = 0.21 than for u_{p0} = 0.4, u_{n0} = 0.2 up to a certain value of η and after that it does not follow so. Again, the values of ϕ_2 are increasing for u_{p0} = 0.3, u_{n0} = 0.21 than that of u_{p0} = 0.4, u_{n0} = 0.2.

In Fig. 4(b), curves represented by h_1, h_2, h_3, h_4 also show the variation of negative ion concentration [N_{n0} = 0.05, N_{n0} = 0.07] of first and second order compressive solitary waves for non-isothermal plasma when V = 1.5, Q = 4, u_{p0} = 0.4, u_{n0} = 0.2, β =0. 1 and Z = 1.

In the variation of negative ion concentration (N_{n0}) for (He^+, O^-) plasma, the second order soliton solution (ϕ_2) has W-type shape, but ϕ_1 and ϕ_2 have all positive values except for ϕ_2 with V = 1.5, Q = 4, β = 0.1, u_{p0} = .4, u_{n0} = 0.2, N_{n0} = 0.07 and Z = 1 up to a certain value of η, where $|\eta|$< 3 for which some values of ϕ_2 are negative.

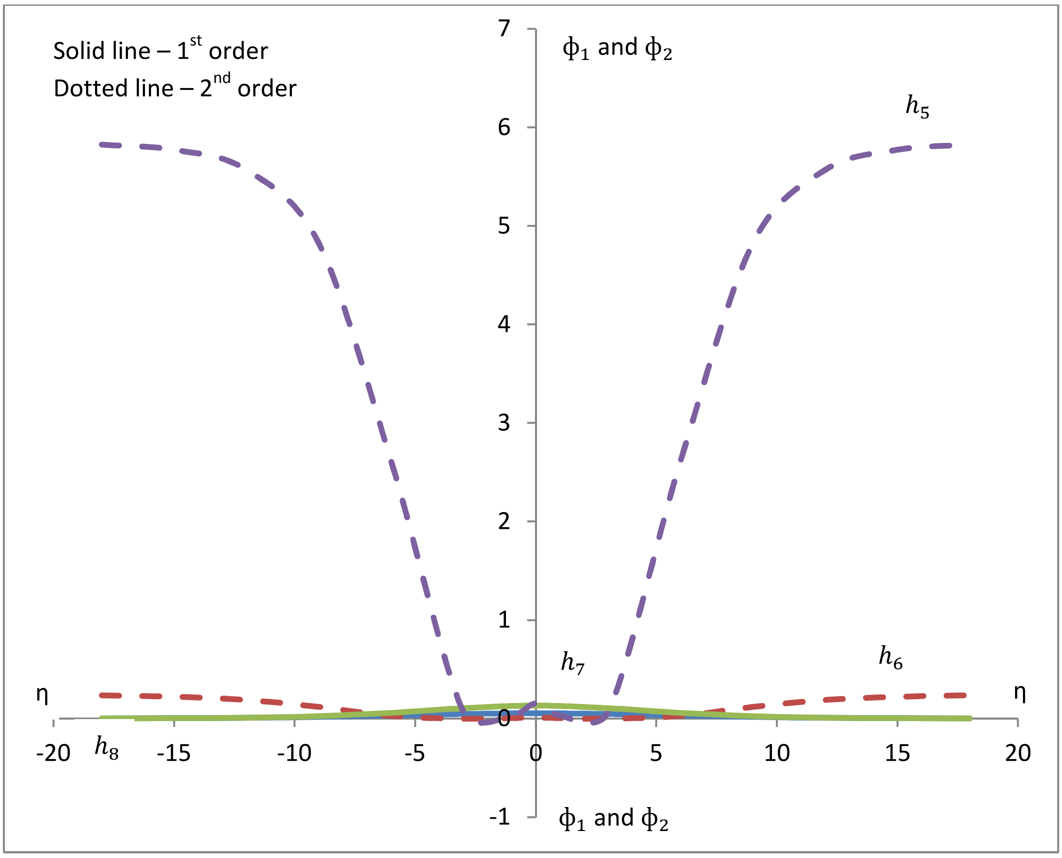

Fig.4(a)

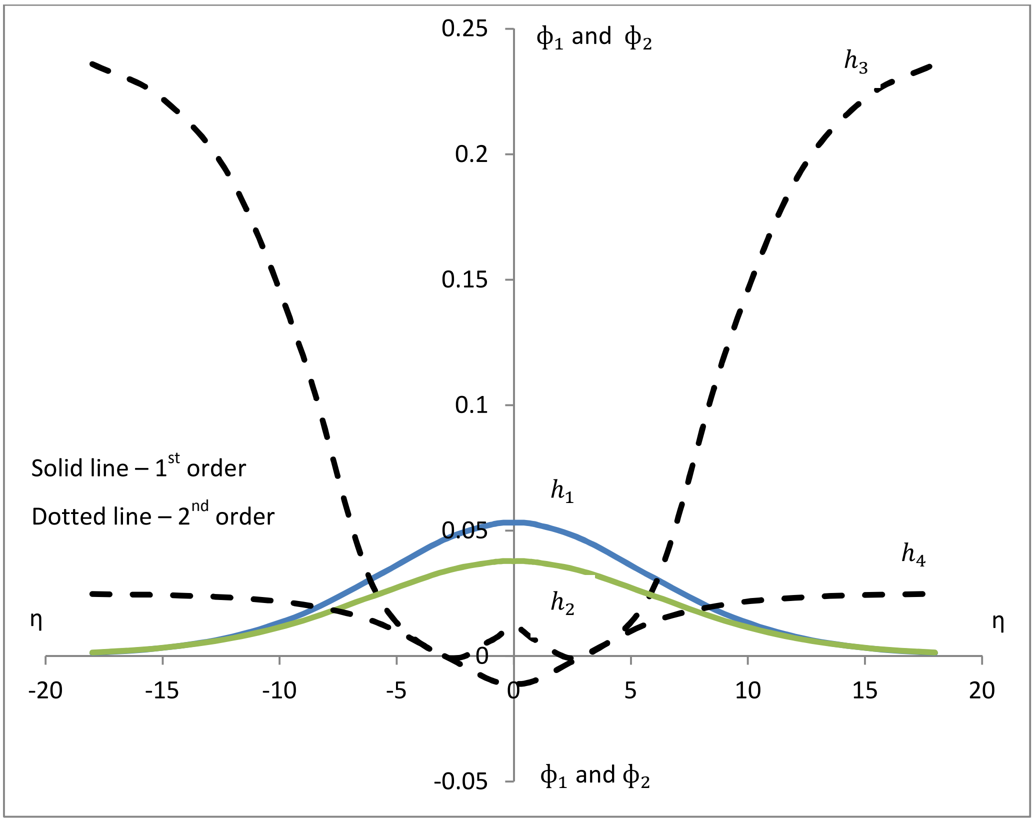

Fig.4(b)

4. Concluding Remarks

We have investigated the ion-acoustic soliton in the plasma consisting of cold positive ions, cold negative ions and warm electrons by pseudo-potential method for both isothermal and non-isothermal cases. Generally, the two types of modes of solitary waves are observed. One is slow and the other is fast ion-acoustic mode. The fast mode nature is obtained here and is discussed with special attention of negative ion concentration (N_{n0}) with respect to that of critical negative ion concentration (N_{nc}). It is very interesting to note that in fast ion-acoustic mode compressive solitary waves are found for $N_{n0} < N_{nc}$ and rarefactive solitary waves are observed for $N_{n0} > N_{nc}$. The first-order (ϕ_1) and second-order (ϕ_2) compressive soliton potentials for the plasma (He^+,O^-) have been found in non-isothermal cases with different plasma parameter variation as shown in Fig. 4. It has an interesting W-type shape. The compressive and rarefactive solitary wave profiles are shown in Figs. 1 and 2 for isothermal case. But in the non-isothermal case, only the compressive solitary wave profiles are found as shown in Figs. 3 and 4. Ichikawa et al. [23] studied the effect of higher order non-linearity on ion-acoustic solitary waves in the two-component plasma consisting of electrons and positive ions. Tagare et al. [21] found the same higher order non-linearity on ion-acoustic solitary waves in a collision-less three-component plasma consisting of negative ions, positive ions with either isothermal or non-isothermal electrons by reductive perturbation method . We have taken the same plasma model of Tagare et al. [21] with the interesting drift concepts with pseudo-potential technique. It is now difficult for us to compare the theoretical result with experiment because no such experiment is known to have been conducted with drift motion and for this plasma model. This work may be on the theoretical investigation of conduction of solitary waves specially in conducting solid medium that can be thought as the plasma medium under a special condition. This type of work on the propagation of ion-acoustic solitary waves through a conducting wire may lead to some tremendous application advantages in signal communications. We now plan to study the ion-acoustic solitary waves at critical density of the negative ions with temperature in isothermal and non-isothermal electron plasma with their respective drifts.

Acknowledgments

One of the authors, S. Chattopadhyay, would like to thank Dr. D. Ray for his valuable suggestions and discussions in the preparation of this paper to its present form.

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Received: 1 March, 2012
Accepted: 27 June, 2012