A New Intensity Formula Applied to Atoms, Ions and Stars

Bo Thelin*
Solarphotonics HB, Granitvägen 12B, Uppsala, Sweden

This is a summary paper about the absolute intensity method used in atomic, ionic and stellar spectra. In this method optical light sources are used to study absolute intensities of the atomic, ionic and stellar spectra. Using this method it was possible to organize spectral data and determine the electron temperature of atom, ions and stars. The method is based on a new intensity formula in optical emission spectroscopy (OES). Like the HR-diagram, it is possible to organize the luminosity of stars from different spectral classes. It is also possible to determine the mean electron temperature of optical layers (photospheres) of stars as it is for atoms and ions in laboratory plasmas. The mean value of the ionization energies of different elements of a star has shown to be very significant for each star. We also show that the hydrogen Balmer absorption lines in the stars follow this intensity formula.

1. Introduction

The work to be presented in this paper originates from a certain discovery made by the author and Sten Yngström during the 1980s, while working with auroral spectroscopy at the Swedish Institute of Space Physics in Kiruna. The data of interest were obtained by using a computerized and extremely versatile spectrometer system (IDES) considered suitable for the study of spectral intensities.

According to S. Yngström [1], the intensity I is given as

$$I = C\,\lambda^{-2}(\exp(-J/kT))/(\exp(h\nu/kT)-1) \qquad (1)$$

Where, J is the ionization energy, and C is a factor given by transition probabilities, number densities and sample properties. λ and ν are here the wavelength and frequency of atomic spectra. This means that the new intensity formula consists of 4 part: the C-factor, λ^{-2}-part, the J-dependence exp(-J/kT) and the Planck factor 1/((exp(hν/kT)-1).

The author and Sten Yngström have developed several methods of analysis which support this formula. One of these is the absolute intensity method used in this paper. It was presented by us for the first time in [2] and [3], where laboratory plasmas were used. Here, we present the same kind of study for ionic spectra that was presented earlier for atomic spectra. A similar intensity formula for ions will also be presented. Furthermore, we shall present results on stellar spectra, which were presented for atomic- and ionic spectra. As for the atomic and ionic spectra, it was possible to obtain similar linear relationships between luminosity and spectral class of stars. By using this absolute intensity method it was also possible to measure the electron temperature of laboratory plasmas and photospheres of stars.

The purpose of this paper is to compare this new method of analysis to different kinds of spectra from earlier papers and collect them in one paper. It also includes new data analysis of the hydrogen Balmer lines for different kinds of stars and classes. The analysis of the Balmer hydrogen lines in this investigation satisfies this intensity formula.

2. Atomic Spectra

The intensities in this paper are derived from arc measurements and as given in [4], where the new intensity formula was used in the development of this method of analysis. In that analysis, $\ln(I\lambda^2)$ was plotted versus

$$h\nu\,(1+\theta/h\nu\,\ln(1-\exp(-h\nu/\theta)))\text{ eV}$$

for 17 elements.

Each intensity value is the mean value of many individual values. For each element, the maximum between the difference, i.e., $\ln I = \ln I\lambda^2 - \ln\lambda^2$ was determined. By using these values, $\ln(I_{max}\,\lambda^2_{max})$ was plotted versus 1.6 $J/h\nu_{max}$ leading to a linear graph, which is very important to follow the equation

$$\ln(I_{max}\,\lambda^2_{max}) = \text{const.} - 1.6\,J/h\nu_{max} \qquad (2)$$

This equation is a linear formula, which is used in this paper for atoms, ions and stars.

*btn602003@yahoo.se

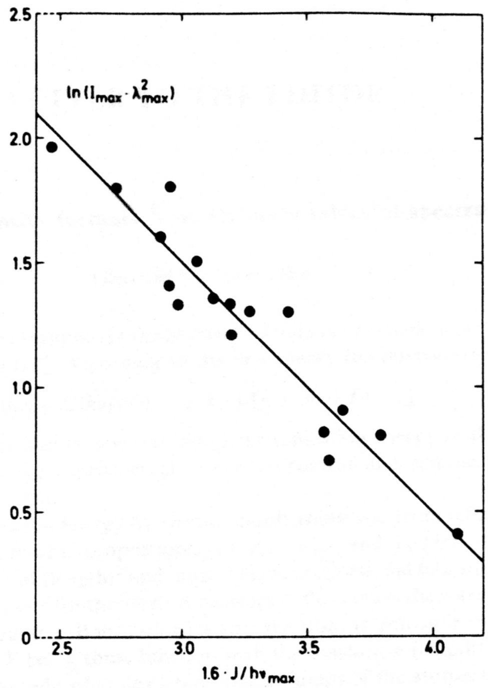

Fig.1: $\ln(I_{max}\,\lambda^2_{max})$ plotted versus (1.6 J)/$h\nu_{max}$ for seventeen elements from the NBS tables in [4].

This graph can be seen in Fig 1, where $\ln(I_{max}\,\lambda^2_{max})$ has been plotted versus 1.6 J/$h\nu_{max}$ = J/θ for 17 elements. Here, θ = k T_e (electron temperature) and J denotes table value of the ionization energy. This graph forms a good linear relationship, where $h\nu_{max}$ = 1.6 θ. The biggest difference between the curves between ln $I\lambda^2$ and ln λ^2 is shown when $h\nu_{max}$ = 1.6 θ. This graph supports the intensity formula.

It was also possible to measure the internal electron temperature for different elements. It is now shown to be possible to obtain similar linear relationships when using intensity data of stellar optical spectra. In Table 1, the electron temperature and ionization energy values from 17 elements are shown. The mean value of these electron temperature values are around 2 eV, which fits well with the values given in[5], where the energy spectra of secondary electrons have been studied. The energy peak for these secondary electrons is situated around 2 eV for atoms, which is in agreement with this paper.

Further evidence in support of this intensity formula has recently been published in [6] and [7] where our method as well as other methods were used.

Table 1. Internal electron temperature (around 2eV) for different elements.

Element	J (eV)	θ (eV)
Cs	3.89	1.6
Na	5.14	1.9
Ba	5.20	1.8
Li	5.39	1.8
Ca	6.11	2.1
Yb	6.25	2.1
Sc	6.70	2.1
Cr	6.76	2.3
Ti	6.83	2.1
Sn	7.33	2.1
Mo	7.38	2.3
Mn	7.43	2.3
Ag	7.57	2.1
Ni	7.63	2.1
Fe	7.86	2.1
Co	7.88	2.2
Pt	9.0	2.1

3. Ionic Spectra

The intensity formula for ions has a similar appearance as Eqn. (1) and is shown in Eqn. (3). This formula includes ionization energies for the first (J_1) and second (J_2) ionization energies, which has been proposed earlier in the detection limit method [10] and in [8] and [9]. The J-term in Eqn. (1) represents the most probable energy in the term scheme because it represents the mean value (or very near) of the energy levels where the energy levels converge against the ionization energy in the term scheme. The same thing appears for ions where the sum (J_1 + J_2) represents the most probable ionic energy levels. C is a factor given by transition probabilities, number densities and sample properties. Here, λ and ν are the wavelength and frequency of the ionic spectral line, respectively. The ionic intensity formula has the following appearance:

$$I = C\,\lambda^{-2}(\exp(-(J_1+J_2)/kT))/(\exp(h\nu/kT)-1) \quad (3)$$

To show the validity of Eqn. (3) by this method, $\ln(I\,\lambda^2)$ was plotted versus $h\nu(1+\theta/h\nu\,\ln(1-\exp(-h\nu/\theta)))$ eV for 11 elements. Each intensity value comes from the NBS table of [4]. By forming the maximum of the difference between ln $I\lambda^2$ and ln λ^2, $\ln(I_{max}\,\lambda^2_{max})$ was plotted versus $1.6(J_1+J_2)/h\nu_{max}$ =(J_1+J_2)/θ in the same way as for atoms as can be seen above. The points follow an expression in

Eqn. (4) for ions, which is similar to Eqn. (2) for atoms

$$\ln(I_{max}\,\lambda^2_{max}) = const. - 1.6(J_1+J_2)/h\nu_{max} \qquad (4)$$

Eqn. (4) includes the first and second ionization energies. For 11 ionic elements, (J_1+J_2) and θ are given in Table 2. These values fit well with the secondary electron temperature values from the literature in [5]. In [5], the energy spectra of secondary electrons have been studied. The energy peak for these secondary electrons is situated around 4 eV for ions, which is in accordance with this paper. For ions the converging levels is the sum of the ionization energies $(J_1 + J_2)$, which represent the most probable ionic energy level.

A graph similar to Fig. 1 for atoms has also been plotted for ions of 11 elements (Fig. 2). This graph shows a good linear relationship according to the linear Eqn. (4), where θ is determined from the linearity and the $h\nu_{max} = 1.6\,\theta$ expression. A more profound explanation of this method of analysis can be seen in [3] on p.1571. The mean value of electron temperatures is about 4 eV for ions, which fits well with the values given in [5].

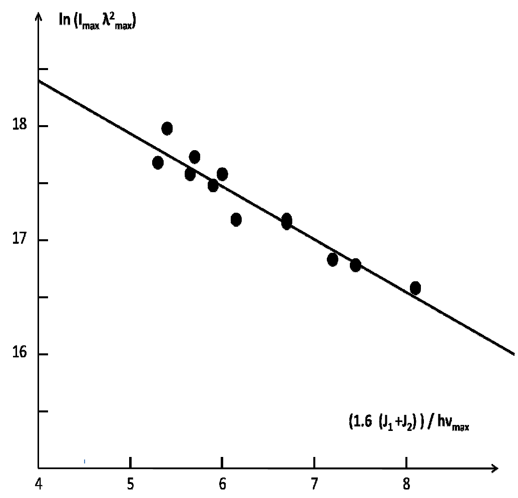

Fig.2: $\ln(I_{max}\,\lambda^2_{max})$ plotted versus $(1.6(J_1 + J_2))/h\,\nu_{max}$ for eleven ionic elements from the NBS tables.

Table 2: Values of (J_1+J_2) and θ of elements.

Element	$(J_1 + J_2)$ (eV)	θ (eV)
Yb	18.36	3.3
Y	19.00	3.3
Sc	19.60	3.6
Ti	20.44	3.4
Mn	23.07	4.4
Cr	23.46	3.5
Fe	24.10	4.2
C	35.65	4.9
K	36.15	4.5
Cs	36.35	5.1
Cu	28.0	5.0

According to [8] and [9], a general recursion formula for two adjacent ionic states (r and r+1) could be written as follows

$$I_{r+1} = C_r\,\lambda^{-2}(\exp(-(J_r+J_{r+1})/kT))/(\exp(h\nu/kT)-1) \qquad (5)$$

4. Stellar Spectra

These stellar optical spectra extend over the spectral classes O–M and the photo-metrically well calibrated luminosity measurements from star to star, and come from [11]. This reference is a digital optical stellar library, which is important in astrophysics. A big part of the data was measured at Kit Peak National Observatory using a 2.3m telescope in combination with a B&C spectrograph and a JHCLIB spectrophotometer.

Good temperature, luminosity and wavelength coverage have been achieved. The data were digitalized from the main sequence classed O5–F0 and F6–K5 displayed in term of relative flux as a function of wavelength. The parameters that have been measured in this investigation are maximum luminosity L_{max} (Rel.fluxmax) of the Planck curve. In these measurements, the maximum wavelength λ_{max} and the maximum frequency ν_{max} were also measured. The hydrogen Balmer absorption lines were also measured.

Then, ln $(L_{max}\,\lambda_{max}^2)$ values were plotted versus $(1.6\,J_{meanvalue}/h\nu_{max})$, where $J_{meanvalue}$ is the mean value of the ionization energies of the elements of the stars measured. To obtain a similar linear relationship for the stellar data as in Fig. 1 and from the spectroscopy method [2] and [3], the following luminosity data from [11] and data from

Table 3 were used and plotted according to Eqn. (6),

$$\ln(L_{max}\,\lambda_{max}^{2}) = \text{const.} - (1.6\,J_{meanvalue}\,h\nu_{max}) \quad (6)$$

This is similar to Eqn. (2) for atoms and Eqn. (4) for ions.

To obtain values given in Table 3, it is necessary to use a two step procedure. In the first step, it is necessary to define the graph by calculating the $J_{meanvalue}$ of the G2-star. The $J_{meanvalue}$ can be expressed in the following way:

$$J_{meanvalue} = \sum c_n J_n \quad (7)$$

Where, c_n is the normalized content of an element of a star. It is plausible to consider the content values of G2-stars similar to the content values of our Sun. Therefore, the c_n-values of the Sun have been used here. J_n is here the mean ionization energy of the elements of a star. This $J_{meanvalue}$ has been calculated for the Sun (G2 star), which gave $J_{meanvalue}$ = 16.2 eV according to the linear graph in Fig 3. This value is 16.2 eV, too, for the sun when using Eqn. (6) together with established chemical composition values of the sun. This means that we now have one point determined in Fig. 3 (step 1). The $J_{meanvalue}$ for each spectral class is calculated by satisfying thevalue, $h\nu_{max} = 1.6\ \theta$ expression, the linearity of Eqn. (6) and the sun point as shown in Table 3 (step 2).

A detailed description of this method of creating Fig. 3 and Table 3 is described in [12]. Table 3 gives Internal electron temperatures and ionization energy mean values for different spectral classes.

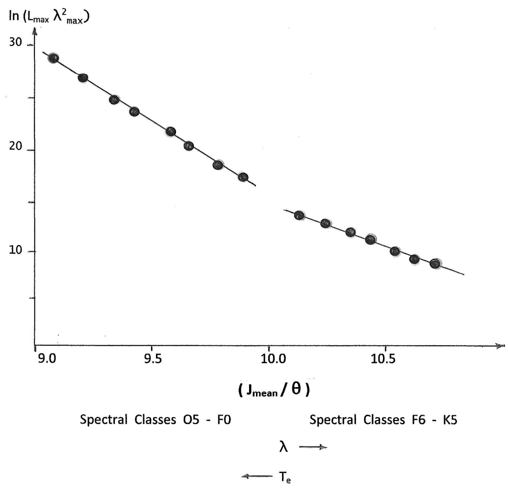

Fig.3: $\ln(L_{max}\,\lambda^{2}_{max})$ plotted versus $(1.6\,J_{meanvalue})/h\nu_{max}$ for different stars from spectral classes O-M.

The data in Fig. 3 constitute a straight line in the classes O5–F0 and F6–K5.

In Eqn. (6), $h\nu_{max} = 1.6\ \theta$, where θ = internal electron temperature in eV. This means that the classes O5–F0 have higher temperature than the classes F6–K5, which is also in accordance with the usual HR-diagram. For example, a G2 star (the sun) has θ = 1.56 eV(T_e = 18110 K).

5. The Use of the Balmer Lines

It is shown in [11] that the appearance of continuous and discrete spectra of stars seems to be the same, where the hydrogen Balmer absorption lines of different stars have been studied. These are the well-known Planck curves with steep low wavelength side and a slow high wavelength side. The wavelength of the intensity maximum of continuous and discrete spectra seems to be the same. This means that the intensity maximum of the Planck curves (continuous spectrum) and the intensity maximum (deepest absorption lines) of the discrete absorption spectrum have about the same λ-value and intensity profile. This is in agreement with Eqn. (1) and our theory in which the Planck factor is a part of the intensity formula. This is clearly seen in Fig. 4 from the spectra of two A-stars. The normalized flux is here proportional to the emissions from the continuous and discrete spectra. These are good examples of Planck curves, where continuous and discrete emissions seem to have the same wavelength maximum.

The wavelengths of the Balmer lines are shown in Table 4 (Ref. 13). By using Eqn. (1) and Table 3, intensity ratios have been determined theoretically (from intensity formula) and by using the experimental data of [11] from different spectral classes of stars. These intensity ratios J_H = 13.595 eV for hydrogen were used. The electron temperatures for different spectral classes have earlier been determined in Table 1 of [12]. A summary of the values from the spectral classes of this paper is shown in Table 5. Good correlation (r = 0.98) has been achieved between theoretical and experimental intensity ratios.

This is shown in Fig. 5, which together with Fig. 4 is a strong evidence of the fact that stars follow the new intensity formula, as atoms and ions do. Fig. 5 shows very good correlation between the experiment and theory.

Table 3: Internal electron temperatures and ionization energy mean values for different spectral classes.

Spectral class	$J_{meanvalue}$ (eV)	θ (eV)
K5V	15.5	1.44
K4V	15.6	1.47
G9-K0	15.8	1.50
G6-G8	16.0	1.53
G1-G2	16.2	1.56
F8-F9V	16.7	1.63
F6-F7V	16.5	1.63
A9-F0V	16.9	1.72
A8	17.1	1.75
A5-A7	17.5	1.81
A1-A3	17.6	1.84
B6V	17.8	1.88
B3-B4V	18.0	1.94
O7-B0V	18.1	1.97
O5V	18.2	2.00

Table 4: Balmer lines used here.

H_α	6562.80 Å
H_β	4861.32 Å
H_γ	4340.46 Å
H_δ	4101.73 Å
H_ε	3970.07 Å

Table 5: Spectral classes and mean electron temperature.

Spectral class	θ (eV)
A8	1.75
A5-A7	1.81
A1-A3	1.84
B6V	1.88
B3-B4V	1.94

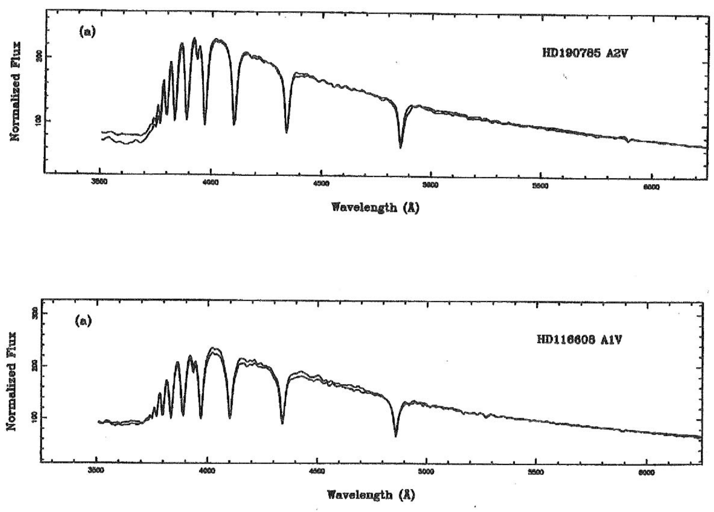

Fig.4: Plot of normalized flux versus the wavelength (Planck curve) for two different A-stars. The absorption hydrogen Balmer lines are clearly observed. The wavelength of the intensity maxima for both continuous and discrete emissions seems to be the same.

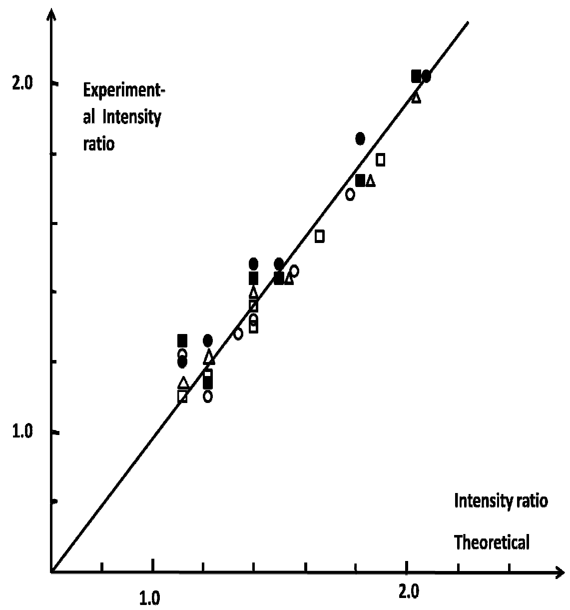

Fig.5: Spectral intensity ratios (experimental and theoretical) give very good correlation(r = 0.98) using the Balmer lines from different spectral classes of stars using the new intensity formula.
Spectral classes used: A8 = unfilled circles, A5-A7 = unfilled squares, A1-A3 = unfilled triangles, B6V = filled circles, B3-B4 = filled squares.

6. Discussion

This method of analysis has shown to be a simple method of verifying the new intensity formula by using atomic, ionic- and stellar data. Using this method linear relationships in the range up to 30 eV for atoms and up to 200 eV for higher ions, have been obtained in [8], with correlation coefficients of 0.90-0.99.

By using this method, together with the new intensity formula, it has been possible to determine the mean electron temperature in different laboratory plasmas and in the optical layers of a star without knowing so much about the chemical composition of the star. These mean electron temperature values fit well with values obtained by other methods given in the literature. The method also gives an organizing method for stars similar to the established HR-diagram. The $J_{meanvalue}$ has shown to be a kind of “signum” for every star. Fig. 3 has shown to be a valuable and simple method of organizing and classifying the stars without knowing so many other details about the stars. The Balmer spectral absorption lines seem to follow the new intensity formula too, which is clearly seen in Figs. 4 and 5. This is clearly seen by the correlation coefficient 0.98. This means that discrete emissions in the star do follow the new intensity formula but are heavily absorbed in the star. Therefore, the light coming from the star is mostly continuous radiation following the Planck radiation law. Fig. 5 also verifies the θ- values of [12] and the θ-values of Table 5 based on Fig. 3, because these were used in the theoretical values of Fig. 5. This means that Fig. 5 verifies Fig. 3 and the new intensity formula.

7. Conclusion

This paper shows that the new intensity formula is a very important formula for the emission of light (atomic and ionic) in optical light sources. It is also important for stars where the hydrogen Balmer absorption lines do follow it.

The absolute intensity analysis method used here for optical light sources turned out to be an important one for an analysis of stars too. By this method, the mean electron temperatures of different light sources and of the photospheres of stars have been calculated.

These values are in accordance with other methods. It is also possible to determine J_{mean}, which has shown to be a “signum” for each star. This method shows that it is possible to organize luminosity data in a linear way as the HR-diagram. This linearization does follow the new intensity formula.

Acknowledgments

I would like to express my gratitude to my colleague and friend Dr. Sten Yngström, at the Swedish Institute of Space Physics, for valuable and interesting discussions about this work.

References

[1] S. Yngström, Int. J. Theor. Phys. **33**, No.7, 1479 (1994).
[2] B. Thelin and S. Yngström, Spectrochim. Acta **41B**, 403 (1986).
[3] S. Yngström and B. Thelin, Appl. Spectrosc. **44**, 1566 (1990).
[4] W. F. Meggers, C. H. Corliss and F. Scribner, Tables of Spectral Line Intensities, National Bureau of Standards, Monograph **32,** Part 1, Washington D.C. (1961).
[5] M. Kaminsky, *Atomic & Ionic Impact Phenomena on Metal Surfaces* (Springer-Verlag, Berlin, Heidelberg, New York, 1965).
[6] B. Thelin, Eurasian J. Anal. Chem. **4**(3), 226 (2009).
[7] B. Thelin, African Phys. Rev. **4**, 121 (2010).

[8] B. Thelin, Indian J. Pure & Appl. Phys. **50**, 231 (2012).
[9] B. Thelin, The African Review of Physics **7**, 31 (2012).
[10] S. Yngström, Applied Spectroscopy **48**, 587 (1994).
[11] D. R. Silva and M. E. Cornell, Astrophys. J. Suppliment Series, **81**, No.2, Aug. (1992).
[12] B. Thelin, Fizika B **19**, 4 (2010) 329.
[13] K. R. Lang, *Astrophysical Formula* (Springer-Verlag, Berlin, Heidelberg, New York, 1974).

Received: 12 June, 2012
Accepted: 28 November, 2012