

Bianchi Type - II, VIII and IX Dark Energy Cosmological Models in Saez-Ballester Theory of Gravitation

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Spatially homogeneous Bianchi type - II, VIII & IX dark energy anisotropic as well as isotropic cosmological models with variable equation of state (EoS) parameter are presented in a scalar tensor theory of gravitation proposed by Saez and Ballester (1986). To get a determinate solution of the field equations we take the help of a special law of variation for Hubble's parameter presented by Bermann (1983) which yield cosmological models with negative constant deceleration parameter. Some important features of the models, thus obtained, have been discussed.

1. Introduction

Scalar-Tensor theories of gravitation are considered to be essential to describe the gravitational interactions near the Planck scale string theory, extended inflations and many higher order theories implying scalar field. Saez and Ballester [1] formulated a scalar-tensor theory of gravitation in which the metric is coupled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of weak fields. In spite of the dimensionless character of the scalar field an antigravity regime appears. This theory also suggests a possible way to solve missing matter problem in non flat FRW cosmologies.

The field equations of the Saez-Ballester scalar tensor theory are

$$G_{ij} - \omega \phi^m \left(\phi_{,i} \phi_{,j} - \frac{1}{2} g_{ij} \phi_{,k} \phi^{,k} \right) = -8\pi T_{ij} \quad (1)$$

and the scalar field ϕ satisfies the equation

$$2\phi^m \phi^{,i}_{,i} + m\phi^{m-1} \phi_{,k} \phi^{,k} = 0 \quad (2)$$

Where, $G_{ij} = R_{ij} - \frac{1}{2} R g_{ij}$ is an Einstein tensor, R the scalar curvature, ω and m are constants, T_{ij} is the stress energy tensor of the matter. The equation of motion is

$$T^{ij}_{,j} = 0 \quad (3)$$

The study of cosmological models in the framework of scalar-tensor theories has been the active area of research for the last few decades. In particular, Singh and Agrawal [3], Shri Ram and Tiwari [4], Reddy and Naidu [5] and Rao et al. [6] are some of the authors who have investigated several aspects of the cosmological models in the Saez-Ballester scalar-tensor theory.

Recently, there has been considerable interest in cosmological models with dark energy in General Relativity because of the fact that our universe is currently undergoing an accelerated expansion, which has been confirmed by a host of observations, such as type Ia supernovae [7,8,9]. Based on these observations, cosmologists have accepted the idea of dark energy, which is a fluid with negative pressure making up around 70% of the present universe energy content to be responsible for this acceleration due to repulsive gravitation. Cosmologists have proposed many candidates for dark energy to fit the current observations such as cosmological constant, tachyon, quintessence, phantom and so on. Current studies to extract the properties of a dark energy component of the Universe from observational data focus on the determination of its equation of state $w(t)$, which is the ratio of the dark energy's

pressure to its energy density $w(t) = \frac{p}{\rho}$ and which is not necessarily constant. The methods for restoration of the quantity $w(t)$ from expressional data have been developed [10], and an analysis of the experimental data has been conducted to determine this parameter as a function of cosmological time [11]. Recently, the parameter $w(t)$ has been calculated with some reasoning that

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reduced to some simple parameterization of the dependences by some authors [12,13,14,15,16,17]. The simplest dark energy candidate is the vacuum energy ($w = -1$), which is mathematically equivalent to the cosmological constant (Λ). The other conventional alternatives, which can be described by minimally coupled scalar fields, are quintessence ($w > -1$), phantom energy ($w < -1$) and quintom (that can cross from phantom region to quintessence region as evolved) and have time dependent EoS parameter. Due to lack of observational evidence in making a distinction between constant and variable w , usually the equation of state parameter is considered as a constant [18,19] with phase wise value -1 , 0 , $+1/3$ and $+1$ for vacuum fluid, dust fluid, radiation and stiff dominated universe, respectively. But in general, w is a function of time or redshift [20,21].

For instance, quintessence models involving scalar fields give rise to time dependent EoS parameter w [22,23,24]. Ray et al. [25], Yadav and Yadav [26], Kumar [27] and Pradhan et al. [28] are some of the authors who have investigated dark energy models in general relativity with variable EoS parameter in different contexts.

Bianchi type space-times play a vital role in understanding and description of the early stages of evolution of the universe. In particular, the study of Bianchi type - II, VIII & IX universes are important because familiar solutions like FRW universe with positive curvature, the de Sitter universe, the Taub-Nut solutions etc. correspond of Bianchi type - II, VIII & IX space-times. Also Rao and Sanyasi Raju [29] and Sanyasi Raju and Rao [30] have studied Bianchi type - VIII and IX models in Zero mass scalar fields and self creation cosmology. Rao et al. [31,32,33], have studied Bianchi type - II, VIII & IX various cosmological models in different theories of gravitation. Rao et al. [34] have studied Bianchi type - I dark energy model in the Saez-Ballester scalar tensor theory of gravitation. Naidu et al. [35] have obtained LRS Bianchi type - II dark energy model in a scalar tensor theory of gravitation. Rao and Sreedevi Kumari [36] have discussed a cosmological model with negative constant deceleration parameter in a general scalar tensor theory of gravitation. Recently, Rao et al. [37] have discussed LRS Bianchi type - I dark energy cosmological model in the Brans-Dicke [38] theory of gravitation.

In this paper, we will discuss Bianchi types II, VIII & IX dark energy models with variable EoS parameter in a scalar tensor theory of gravitation proposed by [1] to understand more physical

aspects, especially the effects of the dark energy in the universe.

2. Metric and Energy Momentum Tensor

We consider a spatially homogeneous Bianchi type - II, VIII & IX metrics of the form

$$ds^2 = dt^2 - R^2 [d\theta^2 + f^2(\theta)d\phi^2] - S^2 [d\varphi + h(\theta)d\phi] \quad (4)$$

Where, (θ, ϕ, φ) are the Eulerian angles, R and S are functions of t only.

It represents

$$\begin{aligned} \text{Bianchi type-II} & \quad \text{if } f(\theta) = 1 \text{ and } h(\theta) = \theta \\ \text{Bianchi type-VIII} & \quad \text{if } f(\theta) = \cosh \theta \text{ and } h(\theta) = \sinh \theta \\ \text{Bianchi type-IX} & \quad \text{if } f(\theta) = \sin \theta \text{ and } h(\theta) = \cos \theta \end{aligned}$$

The energy-momentum tensor of the fluid is

$$T_i^j = \text{diag}[T_1^1, T_2^2, T_3^3, T_4^4] \quad (5)$$

We can parameterize this as follows,

$$\begin{aligned} T_i^j &= \text{diag}[-p_x, -p_y, -p_z, \rho] \\ &= \text{diag}[-w_x, -w_y, -w_z, 1]\rho \\ &= \text{diag}[-(w + \gamma), -(w + \gamma), -w, 1]\rho \end{aligned} \quad (6)$$

Where, ρ is the energy density of the fluid, p_x , p_y and p_z are the pressures and w_x , w_y and w_z are the directional EoS parameters along the x , y and z axes, respectively. $w(t) = \frac{p}{\rho}$ is the deviation free EoS parameter of the fluid. We have parameterized the deviation from isotropy by setting $w_z = w$ and then introducing skewness parameter γ , which is the deviation from w along both x and y axes.

3. Solutions of Field Equations

Now with the help of Eqns. (5) and (6), the field equations, Eqn. (1) for the metric given in Eqn. (4) can be written as

$$\frac{\ddot{R}}{R} + \frac{\ddot{S}}{S} + \frac{\dot{R}\dot{S}}{RS} + \frac{S^2}{4R^4} - \frac{\omega}{2}\phi^m\dot{\phi}^2 = -8\pi(w+\gamma)\rho \quad (7)$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2 + \delta}{R^2} - \frac{3S^2}{4R^4} - \frac{\omega}{2}\phi^m\dot{\phi}^2 = -8\pi w\rho \quad (8)$$

$$2\frac{\dot{R}\dot{S}}{RS} + \frac{\dot{R}^2 + \delta}{R^2} - \frac{S^2}{4R^4} + \frac{\omega}{2}\phi^m\dot{\phi}^2 = 8\pi\rho \quad (9)$$

$$\ddot{\phi} + \dot{\phi}\left(2\frac{\dot{R}}{R} + \frac{\dot{S}}{S}\right) + \frac{m}{2\phi}\dot{\phi}^2 = 0 \quad (10)$$

$$\dot{\rho} + 2\frac{\dot{R}}{R}(w+\gamma+1)\rho + \frac{\dot{S}}{S}(w+1)\rho = 0 \quad (11)$$

Here, the overhead dot denotes differentiation with respect to 't'.

When $\delta=0, -1 \& +1$, the field equations, Eqns. (7) to (11), correspond to the Bianchi types - II, VIII & IX universes, respectively.

The field equations, Eqns. (7) to (11), are only four independent equations with six unknowns $R, S, \rho, w, \gamma \& \phi$, which are functions of 't'. Two additional constraints are required to obtain explicit solutions of these field equations.

We solve the above set of highly non-linear equations with the help of special law of variations of Hubble's parameter proposed by [2], which yields constant deceleration parameter of the models of the universe. We consider the constant deceleration parameter of the model defined by

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = \text{constant} \quad (12)$$

Where, $a = (R^2 S f(\theta))^{1/3}$ is the overall scale factor. Here the constant is taken as negative so that it represents an accelerating model of the universe.

From Eqn. (12), we get

$$a = (R^2 S f(\theta))^{1/3} = (c_1 t + c_2)^{1/(1+q)} \quad (13)$$

Where, $c_1 \neq 0$ and C_2 are constants of integration. This equation implies that the condition of expansion is $1+q > 0$.

To get the deterministic solution, it has been assumed that the expansion θ in the model is proportional to the shear scalar σ . This condition leads to

$$S = R^n \quad (14)$$

Where, n is an arbitrary constant.

From Eqns. (13) and (14), we get

$$R = \left(\frac{1}{f(\theta)}\right)^{\frac{1}{n+2}} (c_1 t + c_2)^{\frac{3}{(1+q)(n+2)}} \quad (15)$$

$$S = \left(\frac{1}{f(\theta)}\right)^{\frac{n}{n+2}} (c_1 t + c_2)^{\frac{3n}{(1+q)(n+2)}} \quad (16)$$

From Eqns. (10), (15) and (16), we get

$$\phi^{\frac{m+2}{2}} = \frac{m+2}{2} \left(\frac{c_3}{c_1} \frac{(q+1)}{(q-2)} f(\theta) (c_1 t + c_2)^{\frac{q-2}{q+1}} + c_4 \right), \quad q \neq 2 \quad (17)$$

Where, c_3 and c_4 are constants of integration.

3.1. Bianchi type - II ($\delta=0$) cosmological model

From Eqns. (9) and (15) - (17), we get the energy density ρ as

$$\begin{aligned} 8\pi\rho = & \left(\frac{9(2n+1)}{(1+q)^2(n+2)^2} \right) \frac{c_1^2}{(c_1 t + c_2)^2} \\ & - \frac{1}{4(c_1 t + c_2)^{\frac{6(2-n)}{(1+q)(n+2)}}} \\ & + \frac{\omega c_3^2}{2(c_1 t + c_2)^{\frac{6}{(1+q)}}}, \quad (n+2) \neq 0 \end{aligned} \quad (18)$$

From Eqns. (8) and (15) - (17), we get the EoS parameter w as

$$\begin{aligned} w = & \frac{1}{8\pi\rho} \left[\left(\frac{6(1+q)(n+2)-27}{(1+q)^2(n+2)^2} \right) \frac{c_1^2}{(c_1 t + c_2)^2} \right] \\ & + \frac{3}{4(c_1 t + c_2)^{\frac{6(2-n)}{(1+q)(n+2)}}} + \frac{\omega c_3^2}{2(c_1 t + c_2)^{\frac{6}{(1+q)}}} \end{aligned} \quad (19)$$

From Eqn. (7) and Eqns. (15) - (17), we get the skewness parameter γ as

$$\gamma = \frac{1}{8\pi\rho} \left[\left(\frac{3(q-2)(n-1)}{(1+q)^2(n+2)} \right) \frac{c_1^2}{(c_1t+c_2)^2} - \frac{1}{(c_1t+c_2)^{\frac{6(2-n)}{(1+q)(n+2)}}} \right] \quad (20)$$

The metric (4), in this case can be written as

$$ds^2 = dt^2 - (c_1t+c_2)^{\frac{6}{(1+q)(n+2)}} (d\theta^2 + d\phi^2) - (c_1t+c_2)^{\frac{6n}{(1+q)(n+2)}} (d\varphi + \theta d\phi)^2 \quad (21)$$

Thus Eqn. (21) together with Eqns. (18), (19) and (20) constitutes a Bianchi type-II dark energy model with variable EoS parameter in a scalar tensor theory of gravitation proposed by [1], which is almost similar to the model obtained [34].

3.2. Bianchi type-VIII ($\delta = -1$) cosmological model

From Eqn. (9) and Eqns. (15) - (17), we get the energy density ρ as

$$8\pi\rho = \left(\frac{9(2n+1)}{(1+q)^2(n+2)^2} \right) \frac{c_1^2}{(c_1t+c_2)^2} - \frac{(\cosh\theta)^{\frac{2}{(n+2)}}}{(c_1t+c_2)^{\frac{6}{(1+q)(n+2)}}} - \frac{(\cosh\theta)^{\frac{2(2-n)}{(n+2)}}}{4(c_1t+c_2)^{\frac{6(2-n)}{(1+q)(n+2)}}} + \frac{\omega_3^2 (\cosh\theta)^2}{2(c_1t+c_2)^{\frac{6}{(1+q)}}} \quad (22)$$

From Eqn. (8) and Eqns. (15) - (17), we get the EoS parameter w as

$$w = \frac{1}{8\pi\rho} \left[\left(\frac{6(1+q)(n+2)-27}{(1+q)^2(n+2)^2} \right) \frac{c_1^2}{(c_1t+c_2)^2} + \frac{(\cosh\theta)^{\frac{2}{(n+2)}}}{(c_1t+c_2)^{\frac{6}{(1+q)(n+2)}}} + \frac{3(\cosh\theta)^{\frac{2(2-n)}{(n+2)}}}{4(c_1t+c_2)^{\frac{6(2-n)}{(1+q)(n+2)}}} + \frac{\omega_3^2 (\cosh\theta)^2}{2(c_1t+c_2)^{\frac{6}{(1+q)}}} \right] \quad (23)$$

From Eqn. (7) and Eqns. (15) - (17), we get the skewness parameter γ as

$$\gamma = \frac{1}{8\pi\rho} \left[\left(\frac{3(q-2)(n-1)}{(1+q)^2(n+2)} \right) \frac{c_1^2}{(c_1t+c_2)^2} - \frac{(\cosh\theta)^{\frac{2}{(n+2)}}}{(c_1t+c_2)^{\frac{6}{(1+q)(n+2)}}} - \frac{(\cosh\theta)^{\frac{2(2-n)}{(n+2)}}}{(c_1t+c_2)^{\frac{6(2-n)}{(1+q)(n+2)}}} \right] \quad (24)$$

The metric (4), in this case can be written as

$$ds^2 = dt^2 - (\sec h\theta)^{\frac{2}{n+2}} (c_1t+c_2)^{\frac{6}{(1+q)(n+2)}} (d\theta^2 + \cosh^2\theta d\phi^2) - (\sec h\theta)^{\frac{2n}{n+2}} (c_1t+c_2)^{\frac{6n}{(1+q)(n+2)}} (d\varphi + \sinh\theta d\phi)^2 \quad (25)$$

Thus Eqn. (25) together with Eqns. (22), (23) and (24) constitutes a Bianchi type-VIII dark energy model with variable EoS parameter in a scalar tensor theory of gravitation proposed by [1].

3.3. Bianchi type-IX ($\delta = 1$) cosmological model

From Eqn. (9) and Eqns. (15) - (17), we get the energy density ρ as

$$8\pi\rho = \left(\frac{9(2n+1)}{(1+q)^2(n+2)^2} \right) \frac{c_1^2}{(c_1t+c_2)^2} + \frac{(\sin\theta)^{\frac{2}{(n+2)}}}{(c_1t+c_2)^{\frac{6}{(1+q)(n+2)}}} - \frac{(\sin\theta)^{\frac{2(2-n)}{(n+2)}}}{4(c_1t+c_2)^{\frac{6(2-n)}{(1+q)(n+2)}}} + \frac{\omega_3^2 (\sin\theta)^2}{2(c_1t+c_2)^{\frac{6}{(1+q)}}} \quad (26)$$

From Eqn. (8) and Eqns. (15) - (17), we get the EoS parameter w as

$$w = \frac{1}{8\pi\rho} \left[\left(\frac{6(1+q)(n+2)-27}{(1+q)^2(n+2)^2} \right) \frac{c_1^2}{(c_1t+c_2)^2} - \frac{(\sin\theta)^{2/(n+2)}}{(c_1t+c_2)^{6/(1+q)(n+2)}} + \frac{3(\sin\theta)^{2(2-n)/(n+2)}}{4(c_1t+c_2)^{6(2-n)/(1+q)(n+2)}} + \frac{\omega^2(\sin\theta)^2}{2(c_1t+c_2)^{6/(1+q)}} \right] \quad (27)$$

From Eqn. (7) and Eqns. (15) - (17), we get the skewness parameter γ as

$$\gamma = \frac{1}{8\pi\rho} \left[\left(\frac{3(q-2)(n-1)}{(1+q)^2(n+2)} \right) \frac{c_1^2}{(c_1t+c_2)^2} + \frac{(\sin\theta)^{2/(n+2)}}{(c_1t+c_2)^{6/(1+q)(n+2)}} - \frac{(\sin\theta)^{2(2-n)/(n+2)}}{(c_1t+c_2)^{6(2-n)/(1+q)(n+2)}} \right] \quad (28)$$

The metric (4), in this case can be written as

$$ds^2 = dt^2 - (\cos ec\theta) \frac{2}{n+2} (c_1t+c_2)^{\frac{6}{(1+q)(n+2)}} (d\theta^2 + \sin^2\theta d\phi^2) - (\cos ec\theta) \frac{2n}{n+2} (c_1t+c_2)^{\frac{6n}{(1+q)(n+2)}} (d\phi + \cos\theta d\phi)^2 \quad (29)$$

Thus Eqn. (29) together with Eqns. (26), (27) and (28) constitutes a Bianchi type - IX dark energy model with variable EoS parameter in a scalar tensor theory of gravitation proposed by [1].

4. Physical and Geometrical Properties

The spatial volume for the models is

$$V = (-g)^{\frac{1}{2}} = (c_1t+c_2)^{3/(1+q)} \quad (30)$$

since $a = (R^2 S f(\theta))^{1/3}$, which is the average scale factor.

The expression for expansion scalar θ calculated for the flow vector, u^i , is given by

$$\theta = u^i_{;i} = \frac{3c_1}{(1+q)(c_1t+c_2)} \quad (31)$$

and the shear σ is given by

$$\sigma^2 = \frac{1}{2} \sigma^{ij} \sigma_{ij} = \frac{7c_1^2}{2(1+q)^2(c_1t+c_2)^2} \quad (32)$$

The components of Hubble parameter H_1, H_2 are given by

$$H_1 = \frac{\dot{R}}{R} = \frac{3c_1}{(1+q)(n+2)(c_1t+c_2)},$$

$$H_2 = \frac{\dot{S}}{S} = \frac{3nc_1}{(1+q)(n+2)(c_1t+c_2)}$$

Therefore, the generalized mean Hubble parameter (H) is

$$H = \frac{1}{3} (2H_1 + H_2) = \frac{c_1}{(1+q)(c_1t+c_2)} \quad (33)$$

The average anisotropy parameter are defined by

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left(\frac{\Delta H_i}{H} \right)^2 = \frac{2(n-1)^2}{(n+2)^2} \quad (34)$$

Where, $\Delta H_i = H_i - H$ ($i = 1, 2, 3$).

5. Conclusions

In this paper, we have presented spatially homogeneous and anisotropic Bianchi type - II, VIII & IX dark energy models with variable EoS parameter in a scalar tensor theory of gravitation proposed by [1]. The power law solution represents a non singular model where the spatial scale factors and volume vanish at

$t = \frac{-c_2}{c_1}$. We observe that all the models have

no initial singularity at $t = \frac{-c_2}{c_1}$ and the spatial

volume is increasing as time t increases, i.e., all the three models are expanding. The Hubble parameter is zero as t approaches to infinity. The scalar expansion θ and the shear scalar σ^2 tend to infinity at $t = 0$, while they become zero as $t \rightarrow \infty$. Since the mean anisotropy parameter $A_m \neq 0$, the models are anisotropic for $n \neq 1$. If $n = 1$ and $A_m = 0$, then the models will become isotropic. Also, since $1 + q > 0$, the models represent accelerating universe. Therefore, it follows that our dark energy models in the Saez-Ballester theory is consistent with the recent observations of Type – Ia supernovae [7,8].

Finally, we can conclude that our models are accelerating, more general and represent not only the early stages of evolution but also the present stage of the universe.

Acknowledgments

The second author (K. V. S. Sireesha) is grateful to the Department of Science and Technology (DST), New Delhi, India for providing INSPIRE fellowship.

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Received: 8 October, 2012
Accepted: 14 December, 2012