The Velocity of Light in Flat Space-time

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The velocity of velocity in vacuum is measured as 299792458 meters per second within the Earth's gravitational environment. This value is usually approximated to that for gravitationally flat space-time (or free space) on the assumption that any correction for the slowing effect of the Earth's gravity would be minimal. The required correction for the slowing can however be done using tools provided by General relativity. From this, we deduce that while the light transit time over one meter on Earth's surface can be 1/299792458 seconds, in a much flatter space, such light transit time over same distance can be as short as 1/299792458.2087s, in conformity with the General relativity prediction that gravity slows light's transit time. This tiny correction reveals its importance for signal observations over astronomical distances, where light velocity in the flatter space-time approaches 299792458.2087m/s.

1. Introduction

The history of whether and how gravity influences light propagation is over 200 years old and may not be necessary to repeat [1]. This influence is manifested by the bending and slowing of light grazing the surface of a celestial body. Einstein's General relativity theory [2,3] has been used to provide the modern quantitative values for such experimentally observed bending and slowing.

Although, the idea behind the theory of general relativity is well known, only specialists are familiar with the mathematically involving derivations and notations. Even among specialists, differences in interpretation occur due to individual aesthetic preferences. For instance, while some may see the deflection of light as a consequence of a reduction of the speed of light near a massive body, others may prefer to describe the same phenomenon as an increase in the time of passage of light (for example, see [4], p.55). Of course, the beauty of the phenomenon is in the eyes of the beholder as long as the quantitative aspects are not tampered with. What is virtually agreeable to all is that gravity affects light transit time.

Notwithstanding this wide acceptance that gravity influences light transit time and whereas Einstein's theory expressly tells us how this influence can be quantitatively determined, it is commonly overlooked to consider this for the light velocity value determined on Earth. It is a certainty from Einstein's theory, as well as from other similar proposals, that all gravitational fields, including that of Earth, will influence light transit time over a given distance, say of one meter.

Therefore, in elevating our terrestrially determined 299792458 m/s light velocity [5,6] to the value in free space, i.e., where the gravitational influence is absent, caution should be exercised in order not to repeat the historical mistakes of our anthropocentric bias. Recall ideas that our Earth was stationary and was the center of the universe, then that our Sun was special and not an ordinary star, all of which were later found unreasonable and subsequently falsified.

If a universal value for the velocity of light in flat space-time is desired, then the necessary correction to remove the Earth's gravitational contribution to the value obtained terrestrially must be undertaken, and it is this that may rightly be regarded as a universal constant for free space. The common reason for not undertaking this task, which we now do, is that gravity is weak and so the Earth's gravitational effect would only lead to a very tiny correction. This may be true, but as the tools to do the correction are available, why not do this? Fortunately, Einstein's equations are available and permit this and we have a simpler, less mathematically involving form of it for use by non-specialists.

We organize the paper by describing the relevant equations that can be used to correct the gravitational influence on light's transit time in the next section. Then, in Sec. 3 we briefly state our findings. In Sec. 4, we discuss some implications and make predictable consequences for our correction. Concluding remarks are reserved for

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Sec. 5. To ensure that the focus is on the value we obtain for light velocity in flat space-time, we reserve three appendices, A, B and C, where other implied matters are discussed in some further detail.

2. The Relevant Equations

Although, the Earth's gravitational field may have only weakly influenced the obtained value 299792458m/s, it is also possible to "weakly" correct this value to that of free space, using our simple Eqn. (1),

$$[\text{Velocity of light on planet}]^2 = [\text{Velocity of light in free space}]^2 - [\text{Escape velocity on planet}]^2 \quad (1)$$

Before using Eqn. (1), let us first establish its correspondence with the equation of General relativity (GR). The square of escape velocity on a planet, i.e., $2GM/r$, plays a prominent role in Einstein's theory, featuring frequently in various forms and forming the basis for many of its predictions, with G being the gravitational constant ($6.67 \times 10^{-11} Nm^2/kg^2$), M the mass of the celestial body (e.g., a planet) and r, the radius of the celestial body or height in the gravitational field. For example, the deviation of light by gravity in GR is given as $(2/c^2)(2GM/r)$ and the effect of gravity on light transit time as

$$d\tau^2 = dt^2 (1- 2GM/r \cdot 1/c^2) \quad (2)$$

Where, $d\tau$, the proper time is the duration of processes free from gravitational influence as will be obtainable in flat space-time at infinity in the gravitational field, dt, the coordinate time is the duration of processes in the presence of a gravitational field, c is the value of light velocity in *free space* and other terms are as earlier defined.

For easier representation of the durations for processes in the different gravitational environments, we write $d\tau$ as $d\tau_\infty$, and c as c_∞, for the time for processes and the velocity of light in free space, respectively. That is,

$$d\tau_\infty^2 = dt^2 (1- 2GM/r \cdot 1/ c_\infty^2) \quad (3)$$

Rearranging, we have

$$1/dt^2 = 1/ d\tau_\infty^2 (1- 2GM/r \cdot 1/ c_\infty^2) \quad (4)$$

Eqns. (2) - (4) can be used to deduce what the duration of a process in flat space-time will be, knowing quantitatively what the duration of the same process is in the presence of a given gravitational field. It can be seen that as Einstein suggested, durations are shorter in the absence of gravity, and longer in its presence. That is, "*the rate of a clock is accordingly slower the greater is the mass of the ponderable matter in its neighbourhood*" [2], p.97. [The corollary to which is that the rate of a clock is faster, the lesser is the mass of the ponderable matter in its neighborhood].

For the process of light transiting a given distance, we can apply Eqn. (4) to that process by multiplying by the square of that distance. Doing this, we get

$$c^2 = c_\infty^2 (1- 2GM/r \cdot 1/ c_\infty^2) \quad (5)$$

Eqn. (5) quantifies the effect of gravity on the duration for light transit over a given distance and shows its correspondence with Eqn. (1). That is,

$$c^2 = c_\infty^2 - 2GM/r \quad (6)$$

We further draw on Einstein's authority in showing how velocity of light is slowed in a gravitational field relative to flat space-time. In Eqn. (3) of his paper [7], that is,

$$c = c_o (1+\phi/c^2) \quad (7)$$

Einstein explicitly showed how knowing the value of light velocity, c, at a given location of known gravitational potential ϕ, we can calculate the value of light velocity, c_o, at what he calls the origin of coordinates, relatively free from gravitational influence. Here, ϕ is $-GM/r$, and if ϕ^2/c^4 is insignificant ($\sim 10^{-19}$), we get a correspondence with our Eqn. (6), by squaring Einstein's equation.

Armed with a rigid one-meter rod, which we intend to take along to various locations at very slow speed to avoid any possible FitzGerald-Lorentz contractions, we first determine the time taken for light to traverse the one meter under the Earth's gravitational environment. This we found to be 1/299792458 seconds and make this dt. Thus, the velocity of light, c, on our planet Earth by calculating light's transit time over one meter will be 299792458m/s. From Eqns. (2) - (4), we can deduce that the same process will take a shorter time in flatter space-time as suggested by Einstein's theory.

3. Findings

Using the tools provided by the GR-derived equations above, knowing c as 299792458m/s and the escape velocity of Earth as 11.186km/s, we find the velocity of light in flat space-time c_∞ as 299792458.2087m/s. That is, light transit time over one meter in flat space-time is 1/299792458.2087seconds. This is slightly shorter than the terrestrial value but conforms fully to the expected slowing effect predicted by GR that the Earth's gravity must have on light. Taking along our rigid one meter rod by slow transport and now with our quantitative value for light velocity in flat space-time c_∞:

On the Moon, given an escape velocity 2.38km/s, light transit time over one meter = 1/299792458.1906s;

On the Sun's surface, given an escape velocity 617.7km/s, light transit time will be 1/299791821.8369s, giving light velocity c_s = 299791821.8369m/s;

On the event horizon of a black hole with $2GM/r \sim c^2_\infty$, light transit time is infinite and the velocity = 0.0000m/s.

Therefore, in this interpretation of GR and as stated by Einstein, "*the velocity of light is everywhere the same, relative to a local inertial system*", [2], p.98. That is, the absolute value obtained for c depends on the space-time curvature of the local inertial system in which the experimental determination is taking place. It follows that the elevation of an obtained value for light transit time or velocity in a given local inertial system to the status of a global or universal value without first correcting for the peculiar influence of that inertial system contradicts GR.

It is inconceivable that Earth's curvature is unique or the flattest possible of an inertial system. As shown above, the Moon's curvature is flatter while the Sun's space-time is more curved. Indeed, to buttress what is interpreted here, Einstein encourages that we discard this anthropocentric bias, by further stating in his book [3] "… according to the general theory of relativity, the law of the constancy of the velocity of light in vacuo, which constitutes one of the two fundamental assumptions in the special theory of relativity and to which we have already frequently referred, cannot claim any unlimited validity. A curvature of rays of light can only take place when the velocity of propagation of light varies with position. Now we might think that as a consequence of this, the special theory of relativity and with it the whole theory of relativity would be laid in the dust. But in reality this is not the case. We can only conclude that the special theory of relativity cannot claim an unlimited domain of validity; its results hold only so long as we are able to disregard the influences of gravitational fields on the phenomena {e.g. of light}" [3], p.89.

This is supportive of our task here of removing the Earth's gravitational influence from the measured light transit time over one meter, in order to increase the domain of validity of the value of light velocity in free space.

4. Implications

Although corrections to the terrestrially determined value of light velocity are relatively small, the following implications can be expected:

(i) Observed bending and slowing of light are already well-known GR predicted outcomes, so we shall not dwell on them. However, what is not well appreciated is that in a reverse direction such as for light travelling from a region of higher gravitational field intensity to a less intense one, as would happen for example when the light grazing the Sun's surface exits, the light will be *unbent* when leaving, again due to changing speed. This follows from the *principle of reversibility of light*. Without this principle inconsistencies would occur in the light paths when viewed in reverse.

To illustrate this, consider two-way signals sent to Venus, while at superior conjunction, grazing the Sun's surface midway. The signals returning to Earth are observed to be bent towards the Sun before reaching Earth. When viewed in reverse, we see that that bending due to the Sun's gravity would be absent in the reverse direction. That is, the light path in reverse from Earth to Venus, grazing the Sun midway is not bent by its encounter with the Sun's gravity, although it is slightly straightened up on leaving the Sun towards Venus (see red line in Fig. 1a). Both the principle of reversibility of light and that of the gravitational effect on light will therefore appear to be violated for this reverse path.

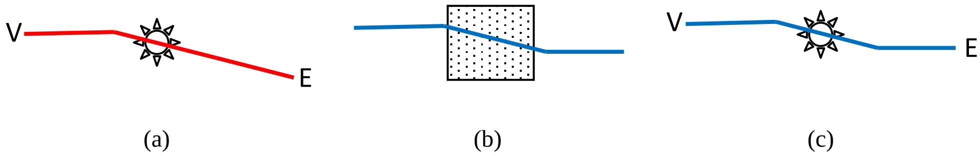

Fig.1: Showing the implications for consistency in interpreting gravitational light bending when the principle of reversibility of light is taken into account (V = Venus, E = Earth).

However, if on exiting the Sun some correction occurs with some straightening up and unbending on exit, just as what happens when light passes through and emerges from a lens (Fig. 1b), then the bending of light by the Sun's gravity is seen along the light's path both in the inward and reverse light paths (see blue line in Fig. 1c). Both the principle of reversibility of light and that of the gravitational effect on light are therefore obeyed by the blue light path in this latter case.

The importance of this principle of reversibility of light to gravitational lensing is that in quantitatively evaluating the experimental evidence for directional shift and time delay, the amount of bending of the light path that is seen on Earth would be a fraction of what actually occurred due to gravity but which is slightly undone on exiting the Sun and travelling towards the Earth-based observer. (See Appendix A for more discussion on bending and Appendix B for a discussion on slowing).

(ii) On a planet, since gravitational potential, $-GM/r$ varies with height on the planet, Doppler frequency effects due to changing light speed may manifest. This is the gravitational frequency shift, also a prediction of GR, viz. "*We therefore conclude that spectral lines which are produced on the Sun's surface will be displaced towards the red, compared to the corresponding lines produced on the earth, by about $2{\cdot}10^{-6}$ of their wavelengths…*" [2], p.97. It is noteworthy that the velocity of light on the Sun's surface that we deduced in section III, i.e., 299791821.8369m/s is also $2{\cdot}10^{-6}$ times less compared to the corresponding velocity on the Earth's surface. Given the known relation between velocity, frequency and wavelength, i.e., $c = f\lambda$, our interpretation quantitatively agrees in many respects with what the author of relativity himself had in mind.

(iii) For light traversing flatter space-time far from Earth, further interesting things may manifest. Signals sent to spacecraft travelling from Earth towards the outer solar system will travel at speeds slightly higher than terrestrial c in parts of the outward and inward journey back to the terrestrial receiver, due to the flatter space-time encountered. If this higher speed is not taken into consideration in computing data received from the receding spacecraft, the expected red shift of signals would appear bluish in coloration and result in anomalous interpretation. The spacecraft would seem to be decelerating and not receding fast enough, as the light signals would be catching up with it and returning to receiver earlier than their scheduled times, which are based on the terrestrial value of c. Furthermore, as $-GM/r$ continues to increase towards the potential at infinity, taken as zero by convention, this discoloration is not constant but gets increasingly bluer with time as the maximal value of signal velocity in absolutely flat space-time is approached.

A rough estimate of the effect of this excess 0.2087m/s of the velocity of light in flat space-time c_∞ over the terrestrial value of light velocity c reveals a red shift much less than what would have been expected using the terrestrially determined velocity 299792458m/s (see Appendix C). Even though this is actually neither deceleration of the spacecraft outward nor acceleration towards the Sun, if it is however desired to interpret it as such, the effect simulates acceleration of about 8.626 x 10^{-10} m/s^2 towards the centre of the solar system (see Appendix D).

We find what is implied here answers "pretty nearly" to the Pioneer spacecraft anomalous observations [8]. Therefore, rather than being a violation of the inverse square law of gravitation or a fault in the equations of general relativity, it is suggested that this is a result of a wrong interpretation of signaling speed. We note here that the anomaly is claimed to have been laid to rest by the JPL team who discovered it, attributing it now to a thermal origin [9]. It is therefore a prediction of this interpretation of general relativity that the same findings and of similar magnitude will resurface in future expeditions, even when the heat loss is eliminated by better engineering.

5. Concluding Remarks

In order not to cloud our focus, what we discussed here is the importance of deriving the value of light velocity in flat space-time and differentiating it from that in local terrestrial space, which by any consideration cannot be said to be flat, even if it is only slightly curved. Other matters concerning whether the value can be relative to the frame of motion of the observer or not are best left to special relativity and other rival theories.

The value we have derived for light velocity in flat space-time is 299792458.2087m/s, using general relativity equations. This weak correction to the terrestrial value would produce predictable and measurable consequences in observations over astronomical distances.

Acknowledgments

I thank Peter Jackson for agreeing to look this paper over and providing some further useful insights and references on the difficulties associated with interpreting and quantifying signal retardation from planetary ranging.

Appendix A

In our interpretation, we view the bending and slowing of light by the Sun's gravity as following the same principle that light obeys when its transit time or velocity changes. This we believe is in agreement with Einstein's reasoning as quoted above. To obtain quantitative values, which astronomers can further compare to general relativistic prediction, we first obtain a refractive index, η for light grazing the Sun's surface as:

η = velocity of light in flat space-time c_∞ divided by velocity of light in Sun's space-time c_s for values
c_∞ = 299792458.2087
c_s = 299791821.8369m/s.

This gives us η = 1.000002123.

We can also obtain same value using Eqn.(5),

$$\eta = 1/\sqrt{(1- 2GM/rc_\infty^2)}$$

For light grazing the Sun's surface, angle of incidence i is 90^o.
Since, $\sin i/\sin r = \eta$, r = 89.88194^o
and the angle of deviation, $\varphi = i - r = 90^o - 89.88194^o = 0.11806^o$ = 425 arc seconds.

This amount of deviation is the magnitude due to the Sun's gravity.

Now for the Sun acting as a lens, we expect a light ray grazing its surface to bend due to gravity and for the light to slightly straighten up again on exit into the flatter space-time before reaching the Earth-based observer, as obtains as well for non-gravitational lenses.

In general relativity, the observed bending of light θ is given as 1.75 arc seconds. In our interpretation, the actual bending φ due to the Sun is much larger, about 425 arc seconds. See the red line and its deviation from the green line in the diagram below.

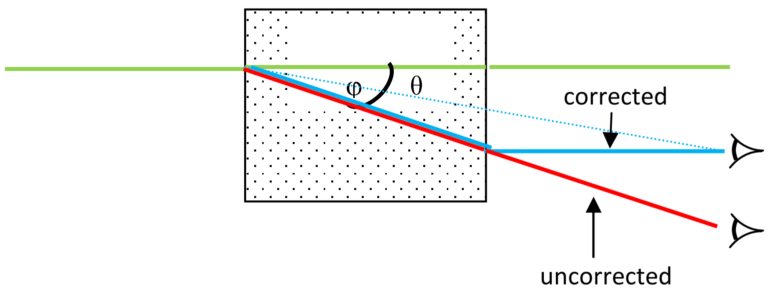

Diagram showing our interpretation of how gravity bends light on entering the field (red line) and how it becomes unbent (blue line) on leaving the gravitational lens, thereby reducing the amount of observable deviation.

On exit from the Sun's intense gravitational field, the correction and unbending that occurs to the light path results in a reduction of observable bending from φ to θ when the light leaves the lens so that only a fraction of the 425 arc seconds is observable from earth. Please see the blue dotted and un-dotted lines in the next diagram.

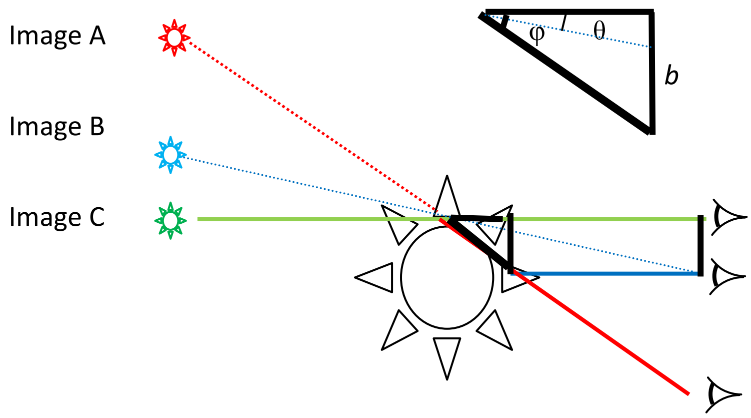

Diagram illustrating the paths of light rays under gravitational influence and the corresponding images A, B and C.

Image **C** is the undistorted image for light unaffected by the Sun's gravity.

Image **B** is the image seen due to the Sun's gravity affecting light grazing its surface but corrected on exit. The amount of deviation θ is smaller than if the light path was not unbent on exit.

Image **A** is what would have been seen due to the light bending but which is not realized because of the subsequent unbending that occurs on exit.

The observed deviation angle φ would have been higher if there was no unbending. Unbending however follows from the combined effect of light on gravity and the principle of reversibility of light. This should be of interest to workers in the field of gravitational lensing.

Taking the bolded right-angled triangle in the diagram for further analysis, from the unbending occurring after the sun's radius has been traversed from the grazing point, we have

tan φ = opp/adj = opposite side b/ Sun's radius ($6.96 \cdot 10^8$m)

Since φ = 0.11806° (425 radians), the opposite side b = 1434784.32m

At the Earth-based observer's location, using the same opposite side 1434784.32m and the adjacent side, now earth distance 1.5 x 10^{11}m, we find the observed angle of bending, after the light path correction on leaving the sun and viewed at earth distance to be ~ 1.97 arc seconds. This is slightly more than what is commonly documented to be 1.75 arc seconds. Astronomical observations and theoretical corrections to our interpretation may bring it closer to an accepted value.

Appendix B

Einstein's attribution of the gravitational bending of light rays to speed change implies that time delay due to the encounter is a consequence. Grazing the Sun's surface in a two-way signaling from Earth, the signals will cross the Sun's diameter (double $6.96 \cdot 10^8$m radius) twice, travelling at c_s (299791821.8369m/s) instead of c (299792458m/s). Travelling at c, the light would have made the diameter in 4.643212205s, but due to the Sun's gravity and travelling at c_s it makes this in 4.643222059s, a difference of $9.854 \cdot 10^{-6}$s. For a two-way trip total delay comes to 19.71μs.

This extra delay equals the non-logarithmic component (i.e., $4GM_s/c^3$ = 19.70μs) in the formula

$$\Delta\tau = 4GM_s/c^3 \cdot (1+ \gamma)/2) \cdot (\ln r_e + r_p + R / r_e + r_p - R)$$

which is usually used in computing the delay, where $\Delta\tau$ is the extra delay, G is the gravitational constant, M_s is the mass of the sun, c is the speed of light, r_e and r_p are the distances from the Sun to the Earth and the target planet, respectively, and R is the distance between the earth and the target planet. In GR, $\gamma = 1$.

For an example of how the formula is used, see Reasenberg et al., 1979, Viking relativity experiment - Verification of signal retardation by solar gravity, ApJ, **234**, L219).

The derivation of the GR formula treats the whole trip as one in Schwarzschild metric, including distances between target planet, Sun and Earth. There are concerns that observed delays may have contributions from planetary clouds (e.g., see J. V. Evans, R. P. Ingalls, 1968, Absorption of Radar Signals by the Atmosphere of Venus, J.

Atmos. Sci. **25**, 555–559) and suggestions that the speed at which gravity propagates may be involved (e.g., see S. M. Kopeikin, 2001, ApJ, **556**, L1 and same author, arXiv:astro-ph/0302462). Our interpretation of the signal retardation is only for the change in speed occurring over the Sun's diameter. It does not include these other theoretical considerations that may be present.

Appendix C

The receding Pioneer spacecraft travelling at about velocity, u_p 12400m/s can no longer be visualized but signals at about frequency, f = 2.3×10^9 Hz are sent to the craft and a transponder sends this back to an Earth receiver for analysis.

Since the spacecraft velocity is very small compared to light velocity, we can find the frequency of returned signal, f' using the classical Doppler equation

$$f' = [c/(c + u_p)] \cdot f$$

rather than the relativistic form of it.

Using terrestrial value of light speed c, 299792458m/s, we can calculate f' as 2299904871.454762Hz, less than the original outgoing signal frequency, a red shift f-f' = 95128.54524Hz.

Using the same signal frequency, same spacecraft velocity but the velocity of light in the flatter space-time traversed, that is
$c_\infty \sim$ 299792458.2087m/s,

f' = 2299904871.454905Hz,

which is still a red shift but this time 0.000143Hz bluer than with the assumption that the velocity of light in the slightly curved space of Earth is the same as that in flat space-time.

Appendix D

As we can no longer see the spacecraft but depend only on returned signals for analysis, a dilemma of interpretation surfaces for the bluish discoloration of the red shift obtained. It may be desired to interpret this as acceleration towards the Sun, even though the spacecraft remains on its trajectory at 12400m/s and may actually neither be decelerating nor accelerating.

As the faster light velocity in flatter space-time, c_∞ catches up with the spacecraft earlier than envisaged, this makes the craft's initial velocity, u_p appear to be slowing to a subsequent velocity, v_p. That is,

$u_p - v_p$ = 0.2087m/s, therefore v_p appears to be 12399.7913m/s.

From the formula $v^2 = u^2 + 2as$, we can write

$$-a = (u_p^2 - v_p^2)/2s$$

Where, s is the distance from the sun. From about 20 A.U. onward (1 A.U. = 1.5×10^{11} m),

$$a = -8.63 \times 10^{-10}\ \text{m/s}^2.$$

This corresponds to the studied anomalous acceleration for the Pioneer spacecrafts ($8.74 \pm 1.33 \times 10^{-8}$m/s^2), if it is desired to interpret the finding as such.

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