Bianchi Type I Cosmological Model with Strange Quark Matter Attached to String Cloud in Self-creation Theory

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In this paper, we investigate the Bianchi type I cosmological model with strange quark matter attached to the string cloud in Barber's second self-creation theory of gravitation. Some physical and kinematical aspects of this cosmological model are also discussed.

1. Introduction

In the very early stages of evolution of our universe it is generally assume that during the phase transition (as the universe passes through it critical temperature) the symmetry of the universe is broken spontaneously as predicted by grand unified theories [1-6]. Moreover, the investigation of cosmic string and their physical properties near such a string has received wide attention because it is believed that the cosmic string gives rise to density perturbation that leads to the formation of galaxies [7]. These cosmic strings have stress energy and coupled to the gravitational field. Therefore, it is interesting to study gravitational effect, which arises from such string by using Einstein's equation. A good many authors have investigated about different aspects of string cosmological models either in the frame work of Einstein's general relativity or in modified theories of gravity. The general relativistic formalisms of cosmic strings are given by Letelier [8] and Stachel [9]. The string cosmologies have been widely studied by [10-23].

It is well known that quark-gluon plasma existed during one of the phase transitions of the universe in the early time when the universe had higher dimensions than it has today and cosmic temperature was T ~ 200 MeV. Studies on the possibility of quark-matter existence date back to the early seventies. Itoh [24], Bodmer [25] and Witten [26] proposed two ways of the formation of quark matter: the quark-hadron phase transition in the early universe and the conversion of neutron stars into strange ones at ultrahigh densities. The quark matter is modelled on an equation of state (EOS) based on the phenomenological bag model of quark matter in which quark confinement is described by an energy term proportional to the volume. In this model, quarks are considered as a degenerate Fermi gas, which exists only in a region of space endowed with vacuum energy density B_c. In the frame work of this model, the quark matter is composed of mass-less u, d quarks, massive s quarks and electrons. In the simplified version of the bag model, assuming that the quarks are mass-less and non-interacting, we have $p_q = \frac{\rho_q}{3}$, where ρ_q is the quark energy density. The total energy density is given by

$$\rho_m = \rho_q + B_c \tag{1}$$

And, the total pressure by

$$p_m = p_q - B_c \tag{2}$$

Mahanta et al. [27] studied the Bianchi type III string cosmological model with quark matter in Barber's Self-creation theory. Rao and Sireesha [28 29] have studied axially symmetric and Bianchi type II, VIII and XI space-time with strange quark matter attached to string cloud, respectively, in the Brans-Dicke theory of gravitation.

In recent years, there have been some interesting attempts to generalize the general theory of relativity by incorporating certain desired features which were lacking in the original theory. Barber [30] proposed two modified theories called self-creation theories. His first theory is a modification of Brans-Dicke [31] theory, whereas the second theory is a modification of the general theory of relativity. Brans [32] pointed out that first theory violates equivalence principle. However due

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to consistency of second theory several authors [33-35] studied various aspects of different cosmological models in Barber's second self-creation theory.

In this paper, motivated by above discussion, we investigate the Bianchi Type I cosmological model with strange quark matter attached to the string cloud in the self-creation theory of gravitation.

2. Metric and Field Equations

We consider the Bianchi type I space-time given by

$$ds^2 = dt^2 - A^2 dx^2 - B^2\left(dy^2 + dz^2\right) \tag{3}$$

Where, A and B are functions of cosmic time t.

The energy momentum tensor for the string cloud is given by

$$T_{ij} = \rho u_i u_j - \rho_s x_i x_j \tag{4}$$

Where, ρ is the rest energy density for the cloud of string with particles attached to them, ρ_s is the string tension density. They are related by

$$\rho = \rho_p + \rho_s \tag{5}$$

Where, ρ_p is the particle energy density. We know that string is free to vibrate. The vibration models of the string represent different types of particles because these models are seen as different masses or spins. Therefore, instead of particles in the string cloud, we take quarks. Hence, we consider strange quark matter energy density instead of particle energy density of the string cloud. In this case, from Eqn. (5), we get

$$\rho = \rho_q + \rho_s + B_c \tag{6}$$

From Eqns. (4) and (6), we have energy momentum tensor for strange quark matter attached to the string cloud as

$$T_{ij} = \left(\rho_q + \rho_s + B_c\right) u_i u_j - \rho_s x_i x_j \tag{7}$$

We have u^i and x^i with

$$u_i u^i = -x_i x^i = 1 \quad \text{and} \quad u_i x^i = 0 \tag{8}$$

We have taken the direction of string along x-axes, then the components of energy momentum tensor are

$$T_1^1 = \rho_s\,, T_2^2 = T_3^3 = 0\,, T_0^0 = \rho \tag{9}$$

The field equation in Barber's second self creation theory is

$$R_{ij} - \frac{1}{2} R g_{ij} = \frac{-8\pi}{\phi} T_{ij} \tag{10}$$

and

$$\phi^k_{;k} = \frac{8\pi}{3}\mu T \tag{11}$$

Where, T is the trace of the energy-momentum tensor, μ is a coupling constant to be determined from the experiment and semi-colon denotes covariant differentiation. In the limit as $\mu \to 0$, this theory approaches the standard general relativity theory in every respect and $G = \phi^{-1}$.

The field equations (10), (11) of self-creation cosmology for the metric in Eqn. (3) with the help of Eqn. (9) can be written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = \frac{8\pi}{\phi}\rho_s \tag{12}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = 0 \tag{13}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = \frac{8\pi}{\phi}\rho \tag{14}$$

$$\ddot{\phi} + \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)\dot{\phi} = \frac{8\pi}{3}\mu\left(\rho + \rho_s\right) \tag{15}$$

Eqns. (12)-(15) are four independent equations in five unknowns A, B, ρ, ρ_s and ϕ. To get indeterminate solution, we assume a relation between metric coefficients given by

$$A = B^n \tag{16}$$

Where, n is a constant.

We consider the deceleration parameter to variable

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = a \text{ (variable)} \tag{17}$$

The above equation may be rewritten as

$$\frac{\ddot{R}}{R} + a\frac{\dot{R}^2}{R^2} = 0 \tag{18}$$

The general solution of Eqn. (18) is given by

$$\int e^{\int \frac{a}{R}dR} dR = t + t_0 \tag{19}$$

Where, t_0 is a constant integration.

In order to solve the problem completely, we have to choose $\int \frac{a}{R} dR$ in such a manner that Eqn. (19) will be integrable.

Without loss of generality, we consider

$$\int \frac{a}{R} dR = \log M(R) \tag{20}$$

Using Eqn. (20) in Eqn. (19), we get

$$\int M(R) dR = t + t_0 \tag{21}$$

The choice of $M(R)$ in Eqn. (21) is quite arbitrary but since we are looking for physically viable models of the universe consistent with observations, we consider the following.

Let us consider

$$M(R) = \frac{1}{\alpha R^m} \tag{22}$$

Where, α and m are arbitrary constant.

Integrating Eqn. (21) and using Eqn. (22), gives

$$R(t) = (bt + c)^{\frac{1}{1-m}} \tag{23}$$

Where, b and c are constants.

Using Eqn. (23), we obtain

$$A = (bt + c)^{\frac{3n}{(1-m)(n+2)}} \tag{24}$$

$$B = (bt + c)^{\frac{3}{(1-m)(n+2)}} \tag{25}$$

From Eqns. (12)-(15), we find

$$\ddot{\phi} + (n+2)\frac{\dot{B}}{B}\dot{\phi} = \frac{8\pi}{3}\mu\left(2(n+1)\frac{\dot{B}^2}{B^2} + 2\frac{\ddot{B}}{B}\right)\phi \tag{26}$$

Which, on simplification gives

$$\phi = \psi_1 (bt + c)^{\frac{-l_1 + \sqrt{l_1^2 + l_2}}{2}} + \psi_2 (bt + c)^{\frac{-l_1 - \sqrt{l_1^2 + l_2}}{2}} \tag{27}$$

Where, ψ_1, ψ_2 are constants of integration and

$$l_1 = \frac{m+2}{1-m} \qquad l_2 = \frac{8\mu(2n + nm + 2m + 4)}{(1-m)^2 (n+2)^2}$$

Thus the geometry of the universe described by the line element with suitable transformation is

$$ds^2 = dT^2 - T^{\frac{6n}{(1-m)(n+2)}} dX^2 - T^{\frac{6}{(1-m)(n+2)}} \left(dY^2 + dZ^2\right) \tag{28}$$

3. Physical and Geometrical Properties of the Model

The physical and kinematical parameters, which play a vital role in the discussion of cosmology, are given as follows.

The Barber scalar ϕ is obtained as

$$\phi = \psi_1 T^{m_1} + \psi_2 T^{m_2} \tag{29}$$

Where, $m_1 = \frac{-l_1 + \sqrt{l_1^2 + l_2}}{2}$ and $m_2 = \frac{-l_1 - \sqrt{l_1^2 + l_2}}{2}$.

As the coupling constant $\mu \to 0$, this theory leads to Einstein's theory and also m_1 vanishes and the other one m_2 does not tend to zero, which is not acceptable and hence the scalar field becomes a constant.

String energy density

$$\rho = \frac{1}{8\pi}\left(\frac{9(2n+1)}{(1-m)^2 (n+2)^2 T^2}\right)\left(\psi_1 T^{m_1} + \psi_2 T^{m_2}\right) \tag{30}$$

String tension density

$$\rho_s = \frac{1}{8\pi}\left(\frac{6mn + 12m - 6n + 15}{(1-m)^2 (n+2)^2 T^2}\right)\left(\psi_1 T^{m_1} + \psi_2 T^{m_2}\right) \tag{31}$$

String particle density

$$\rho_p = \frac{1}{4\pi}\left(\frac{12n-3mn-6m-3}{(1-m)^2(n+2)^2T^2}\right)\left(\psi_1 T^{m_1}+\psi_2 T^{m_2}\right) \tag{32}$$

Quark energy density

$$\rho_q = \rho - B_c = \frac{1}{8\pi}\left(\frac{9(2n+1)}{(1-m)^2(n+2)^2T^2}\right)\left(\psi_1 T^{m_1}+\psi_2 T^{m_2}\right) - B_c \tag{33}$$

Quark pressure

$$p_q = \frac{\rho_q}{3} = \frac{1}{24\pi}\left(\frac{9(2n+1)}{(1-m)^2(n+2)^2T^2}\right)\left(\psi_1 T^{m_1}+\psi_2 T^{m_2}\right) - \frac{B_c}{3} \tag{34}$$

The string energy density, string tension density and string particle density decrease with an increase in T and approaches to zero as T becomes infinite. Also quark energy density and quark pressure are infinite at the initial epoch, and it tends to the bag constant B_c as $T \to \infty$.

Spatial volume

$$V = R^3 = T^{\frac{3}{1-m}} \tag{35}$$

The mean Hubble parameter

$$H = \frac{1}{(1-m)T} \tag{36}$$

Scalar of expansion

$$\theta = 3H = \frac{3}{(1-m)T} \tag{37}$$

The shear scalar

$$\sigma^2 = \frac{3(n-1)^2}{(1-m)^2(n+2)^2T^2} \tag{38}$$

4. Conclusion

We have discussed the Bianchi type I cosmological model with strange quark matter attached to the string cloud in the frame work of self-creation theory. The physical parameters c and σ decreases with the increase in time and approaches to zero as $T \to \infty$. Volume $V \to 0$ when $T \to 0$ and V is infinite as $T \to \infty$. This shows that the model is expanding. We also find that $\frac{\sigma}{\theta}$ tends to constant limit as $T \to \infty$, which shows that the anisotropy in the universe is maintained throughout. However, it become isotropic for $n=1$. The energy condition $\rho \geq 0$ can be fulfilled, provided $-\frac{1}{2} < m < 1$. From Eqn. (32), we can see that the particle density disappears for $n = \frac{2m+1}{4-m}$ and we get only the geometric string model i.e., $\rho_p = 0$ and $\rho = \rho_s$.

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