

Determination of Hubble Constant Using Apparent Magnitude and Redshift of Type-I Supernovae

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The Hubble constant is a very important constant in cosmology which measures the mean expansion rate of the universe. In this paper, we obtain a non-linear relationship between apparent magnitude and redshift of type Ia supernovae (SNe Ia). Using this relation we determine the Hubble constant $H_0 = 63.13 \pm 6.48$ km/s/Mpc.

1. Introduction

From extraordinary evidences it is claimed that we are living in an expanding homogeneous and isotropic universe [1–3]. In 1929, Hubble [4] discovered that the rate at which galaxies are receding from us is proportional to the distance, $v = H_0 r$, where v is the recessional velocity, r is the distance and the proportionality constant H_0 is known as Hubble constant. Almost all models of the universe are based on this relationship, known as ‘Hubble’s law’. The Hubble constant measures the present cosmic expansion rate and is related to the cosmic components and the size and age of our universe [5]. Since the establishment of the linear relationship between the recessional redshift of the galaxies and their apparent distances, accurate determination of Hubble constant has been an age long issue for the cosmologists. The main problem to evaluate the Hubble constant is in the accurate determination of distances to galaxies [3].

For the determination of Hubble constant various methods have been employed over a period of time by different groups. Each method had its own outcome. Some of the results obtained by various methods are as follows: The value of H_0 determined by Hubble himself was $H_0 = 500$ km/s/Mpc with large error. In 1958, Sandage [6] estimated it more precisely as $H_0 = 75$ km/s/Mpc. Using HST, Freedman et al. [7] have obtained $H_0 = 72 \pm 8$ km/s/Mpc, Reiss et al. [8] have obtained $H_0 = 73.8 \pm 1.74$ km/s/Mpc, considering Λ CDM model [9] the nine year WMAP [10] has obtained $H_0 = 70.0 \pm 2.2$ km/s/Mpc and considering the standard flat Λ CDM model, the Planck collaboration [11] has obtained $H_0 = 66.93 \pm 0.62$ km/s/Mpc. A recent measurement by the BOSS collaboration [12] gives $H_0 = 67.6_{-0.6}^{+0.7}$ km/s/Mpc. There are various other

methods which have provided results with good confidence level.

In this paper, we obtain a new relation between the apparent magnitude and redshift for type-I supernovae which is derived without taking any approximation. Using this relation we determine the value of Hubble constant. The paper is organized as follows: In section 2, we derive the relation between apparent magnitude and redshift of type Ia supernovae. In section 3, we determine the value of Hubble constant and compare our result with others.

2. Relation between apparent magnitude and redshift of type Ia supernovae

In this work, we have considered only those type-I supernovae whose redshift is greater than 0.1 ($z > 0.1$). Previously, authors have taken the low values of z . At low redshifts, the data satisfies the Hubble’s law very well [3]. Here, we have carefully chosen the supernovae. The data is taken from SCP Union2.1 compilation project [13,14]. At first we derive a relation between the redshift and the apparent magnitude for type Ia supernovae. While deriving this relationship no approximation is used.

We have the distance-luminosity relation for Einstein-de Sitter universe [15,16] ($k = 0$ and $\Lambda = 0$) as it provides one of the most simple approach to the distance-luminosity, in terms of redshift as:

$$d_L = \frac{2c}{H_0} (1 + z - \sqrt{1 + z}), \quad (1)$$

where z = redshift of the object, c = velocity of the light and H_0 = Hubble constant.

Now we can write the modified expression between m , the apparent magnitude of a standard candle and M with the distance in Mpc as [14,17]

$$m = M + 25 + 5 \log d_L \quad (2)$$

Putting the value of d_L from Eq. (1) in Eq.(2) we get

$$m = M + 25 + 5 \log \left[\frac{2c}{H_0} (1 + z - \sqrt{1+z}) \right] \quad (3)$$

$$\Rightarrow m = M + 25 + 5 \log 2 + 5 \log \left[\frac{c}{H_0} (1 + z - \sqrt{1+z}) \right]$$

$$\Rightarrow m = M + 25 + 5 \log 2 + 5 \log \frac{c}{H_0} + 5 \log [(1 + z - \sqrt{1+z})]$$

$$\Rightarrow \log \frac{c}{H_0} = \frac{1}{5} [m - M - 25 - 5 \log (1 + z - \sqrt{1+z})]$$

$$\Rightarrow \frac{c}{H_0} = \text{antilog} \left[\frac{1}{5} [m - M - 25 - 5 \log (1 + z - \sqrt{1+z})] \right]$$

$$\Rightarrow H_0 = \frac{c}{\text{antilog} \left[\frac{1}{5} [m - M - 25 - 5 \log (1 + z - \sqrt{1+z})] \right]} \quad (4)$$

From equation (4), we can see that there is an explicit relationship between the redshift and the apparent magnitude. The data apparent magnitude and redshift of the SNe Ia for calculating the Hubble constant is taken from SCP “Union2.1” SNe Ia compilation [14]. As previously mentioned only those SNe Ia are taken which have $0.1 < z < 0.9$.

The Table 1 gives a short list of the SNe Ia and their apparent magnitude and redshift that has been used to determine the value. More SNe Ia have been taken into consideration but have not been mentioned in this table.

3. Result and Discussion

The accurate determination of the Hubble constant is a challenging issue in cosmology. So far the value of H_0 is not measured directly. It can only be estimated in the field of a given theory [18]. In this paper, we find that at larger redshifts, the effects of space-time curvature modify the Hubble’s law and there exists a non-linear relationship (Eq. (4)) between the redshift and the apparent magnitude for type Ia supernovae. The inverse of Hubble constant gives the order of magnitude of the age of the universe and multiplied by c gives the order of magnitude of the size of the visible universe [1]. Therefore, the deviations from Hubble’s law at high redshifts can be used to constrain the contents of the universe. Using the data in equation (4) we get the value of Hubble constant $H_0 = 63.13 \pm 6.48$ km/s/Mpc. The measurement by the BOSS

collaboration [12] gives $H_0 = 67.6^{+0.7}_{-0.6}$ km/s/Mpc. The LIGO and Virgo collaboration [19] have measured the Hubble constant $H_0 = 70.0^{+12.0}_{-8.0}$ km/s/Mpc using the distance to the source inferred from the gravitational wave signal and the recession velocity obtained from the measurements of the redshift using the electromagnetic data. In the context of light model, Vigoureux et al. [18] have obtained $H_0 \approx 63$ km/s/Mpc in the whole range $0.1 < z < 8.1$. Recently Silva and Cavalcanti [20] have proposed a new method to estimate H_0 in intermediate redshifts ($z \approx 1$) and obtained $H_0 = 68.00 \pm 2.20$ km/s/Mpc. Our result consistent with the global measurements via CMBR and other recent estimates of H_0 [12,18,20] by different methods.

Our method is different from others because we have used a non-linear relationship between the redshift and the apparent magnitude whereas so far others have used the linear relationship. We hope future experimental as well as theoretical studies will constrain the Hubble constant to high precision.

Table 1: List of the SNe Ia and their apparent magnitude and redshift

Supernovae Name	Redshift (z)	Apparent Magnitude
*03D3cd	0.4607	22.60
04D3kr	0.3373	21.93
*03D4dh	0.6268	23.35
*03D3aw	0.4490	22.54
*D087	0.34	21.79
*D089	0.436	22.46
*D093	0.3630	21.89
*E108	0.4690	22.57
*E140	0.6310	23.32
*E147	0.645	23.32

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