

## Bianchi Type-III Viscous Fluid Models in Bimetric Theory of Gravitation

P. K. Sahoo<sup>1,\*</sup> and B. Mishra

<sup>1</sup>*Department of Mathematics, Birla Institute of Technology and Science-Pilani, Hyderabad Campus, Hyderabad-500078, India*

Spatially homogeneous and anisotropic Bianchi type-III cosmological models are obtained in Rosen's bimetric theory of gravitation [1] when the source of gravitational field is governed by viscous fluid. Various physical and geometrical properties of the models are discussed.

### 1. Introduction

In an attempt to modify the Einstein's general theory of relativity, Rosen [1] proposed a new theory of gravitation known as bimetric theory of gravitation. This theory satisfies covariance and equivalence principles. In this theory, two metric tensors are defined at each point of the space-time: Riemannian metric tensor  $g_{ij}$  and background flat space-time metric tensor  $\gamma_{ij}$ . The tensor  $g_{ij}$  describes the geometry of curved space-time and also the gravitational field (i.e., interaction between matter and gravitation) whereas the tensor  $\gamma_{ij}$  refers to inertial forces (i.e., whose curvature tensor vanishes). Accordingly, at each point of the space-time, one has two line elements

$$ds^2 = g_{ij}dx^i dx^j \quad (1)$$

and

$$d\sigma^2 = \gamma_{ij}dx^i dx^j \quad (2)$$

Since the theory has some noteworthy characteristics, it has excited the interest of many authors to study this theory. The authors who have studied this theory from various angles are Yilmaz [2], Israelit [3,4], Reddy and Venkateswarlu [5], Mohanty and Sahoo [6,7], Reddy et al. [8,9], Sahoo [10-12], and Sahoo and Mishra [13,14]. It is evident from the literature that this theory needs more investigations so as to upheaval the hidden secrets of the theory.

In theoretical cosmology, the Bianchi type cosmologies play an important role. A Bianchi (type) cosmology represents a spatially homogeneous universe as this space-time admits a three param-

eter group of isometries whose orbits are space like hyper-surfaces. These models can be used to analyze the aspects of the physical universe, which pertain to or may be affected by anisotropy in the rate of expansion.

In most cosmological models, the matter in the universe is considered as perfect fluids. But one should expect that viscosity concept may also have an important role in cosmology, particularly in cases where turbulence effects occur. The bulk viscosity in a fluid allows an easy exchange of energy between translational and internal degrees of freedom as in the case of a gas of rough spheres. The viscosity mechanism in cosmology can account for high entropy of the present universe (Weinberg, [15,16]). Bulk viscosity associated with grand unified theory may lead to an inflationary cosmology and introduction of bulk viscosity can avoid the big-bang singularity. Thus we consider bulk-viscosity distribution to have realistic cosmological models (Gron [17]). The solutions found in the model by Murphy [18] exhibit an interesting feature that the big-bang singularity appears in the infinite past. Bali and Upadhaya [19] have discussed Bianchi type-III string cosmological models with bulk viscosity, where the constant coefficient of bulk viscosity is considered. Wang [20-23] have discussed LRS Bianchi type-I and Bianchi type-III model for a cloud string with bulk viscosity and Yadav et al. [24] have studied some Bianchi type-I viscous string cosmological model for cloud of string with bulk viscosity. Rao and Sireesha [25,26] have discussed the Bianchi type-II, VIII and IX string cosmological models with bulk viscosity in various scalar tensor theories of gravitation.

Reddy and Venkateswara Rao [27] have shown the non existence of spatially homogeneous, anisotropic Bianchi type III, V and  $VI_0$  cosmolog-

<sup>1</sup> sahoomaku@rediffmail.com

ical models in bimetric theory when the source of the gravitational field is a perfect fluid. However, they have found vacuum models in bimetric theory. Mohanty and Sahoo [6] have considered the anisotropic spatially homogeneous Bianchi type III and  $V I_0$  metrics in bimetric theory of gravitation with source of gravitation meson field and mesonic perfect fluid and have found vacuum models. Rao et al. [28] have constructed Bianchi type-I string cosmological model with bulk viscosity in bimetric theory of gravitation.

To our knowledge, none of the authors have studied this theory for Bianchi type-III space-time when the source of the gravitational field is governed by viscous fluid. Thus in the present paper, we have taken an attempt to discuss the effect of bulk viscosity in Bianchi type-III space-time.

### 2. Field Equations

The field equations in bimetric theory of gravitation proposed by Rosen [1] are

$$N_j^i - \frac{1}{2}N\delta_j^i = -8\pi\kappa T_j^i \tag{3}$$

where

$$N_j^i = \frac{1}{2}\gamma^{ab}(g^{hi}g_{h|a})_{|b}$$

and

$$N = N_i^i$$

Here a vertical bar (|) denotes the covariant differentiation with respect to  $\gamma_{ij}$  and  $T_j^i$  is the usual stress tensor of the matter fields.

We consider the Bianchi type-III metric in the form

$$ds^2 = -dt^2 + e^{2\alpha}dx^2 + e^{2(\beta+x)}dy^2 + e^{2\delta}dz^2 \tag{4}$$

with the convention  $x^1 = x, x^2 = y, x^3 = z, x^4 = t$  and  $\alpha, \beta$  and  $\delta$  are functions of time  $t$  only.

The flat space-time corresponding to the metric (4) is

$$d\sigma^2 = -dt^2 + dx^2 + e^x dy^2 + dz^2 \tag{5}$$

### 3. Viscous Fluid

In this section, we are interested in constructing viscous fluid models. The energy momentum

tensor for viscous fluid distribution is given by

$$T_j^i = (\rho + \bar{p})u^i u_j + \bar{p}g_{ij} \tag{6}$$

together with

$$u^i u_i = -1 \quad \text{and} \quad \bar{p} = p - \eta u^i_{;j} \tag{7}$$

Where,  $u^i$  is the four velocity vector of the fluid,  $p$  is the proper pressure,  $\rho$  is the energy density,  $\bar{p}$  is the effective pressure and  $\eta$  is the bulk viscous coefficient of the fluid.

Since the bulk viscous pressure represents only a small correction to the thermodynamical pressure, it is reasonable assumption that the inclusion of viscous term in the energy momentum tensor does not change fundamentally the dynamics of the cosmic evolution. For the specification of  $\eta$ , we assume that the fluid obeys an equation of state of the form

$$p = \zeta\rho \tag{8}$$

Where,  $0 \leq \zeta \leq 1$ .

Using co-moving co-ordinate system, the field equations (3) for the metrics (Eqns. (4) and (5)) corresponding to the energy momentum tensor in Eqn. (6) can be written as

$$\alpha_{44} - \beta_{44} - \delta_{44} = 16\pi\kappa\bar{p} \tag{9}$$

$$\alpha_{44} - \beta_{44} + \delta_{44} = -16\pi\kappa\bar{p} \tag{10}$$

$$\alpha_{44} + \beta_{44} - \delta_{44} = -16\pi\kappa\bar{p} \tag{11}$$

$$\alpha_{44} + \beta_{44} + \delta_{44} = 16\pi\kappa\rho \tag{12}$$

Here and afterwards the suffix 4 after a field variable represents ordinary differentiation with respect to time  $t$  only. From the field equations (9)-(11), we obtain

$$\alpha_{44} = \beta_{44} = \delta_{44} = -16\pi\kappa\bar{p} \tag{13}$$

Using the values from Eqn. (13) in Eqn. (12), we get

$$3\bar{p} + \rho = 0 \tag{14}$$

Corresponding to the metric in Eqn. (4), we get from Eqn. (7)

$$\bar{p} = p - \eta[\alpha_4 + \beta_4 + \delta_4] \tag{15}$$

Using Eqn. (15) in Eqn. (14), we obtain

$$3[p - \eta(\alpha_4 + \beta_4 + \delta_4)] + \rho = 0 \tag{16}$$

For reality conditions of  $p$  and  $\rho$ , we must have  $p > 0$  and  $\rho > 0$ . Thus Eqn. (16) satisfies the following cases.

**Case I:**  
When

$$p - \eta(\alpha_4 + \beta_4 + \delta_4) = 0 \tag{17}$$

and

$$\rho = 0 \tag{18}$$

Using Eqn. (17) in Eqn. (15), we get

$$\bar{p} = 0 \tag{19}$$

Again using Eqn. (18) in Eqn. (8), we obtain

$$p = 0 \tag{20}$$

Now, using Eqn. (19) in Eqn. (13), we get

$$\alpha_{44} = \beta_{44} = \delta_{44} = 0 \tag{21}$$

On integration, Eqn. (21) yields

$$\alpha = \beta = \delta = a_1 t + a_2 \tag{22}$$

Where,  $a_1$  and  $a_2$  are constants of integration. Now putting the above values in Eqn. (15), we get

$$\eta = 0 \tag{23}$$

Thus the metric in Eqn. (4) corresponding to Eqn. (22) takes the form

$$ds^2 = -dt^2 + e^{a_1 t + a_2} [dx^2 + e^{2x} dy^2 + dz^2] \tag{24}$$

After using the transformation of coordinates  $a_1 t + a_2 = T, x = X, y = Y$  and  $z = Z$ , the metric (24) can be written as

$$ds^2 = -dT^2 + e^{2T} [dX^2 + e^{2X} dY^2 + dZ^2] \tag{25}$$

The vacuum model of Eqn. (25) is spatially homogeneous and isotropic. The model has no singularity at  $T = 0$ . It is interesting to note that this model is similar to the vacuum model obtained by Reddy and Venkateswarlu [27]. (Hence the work done in this paper may be considered as an extension of the work by Reddy

and Venkateswarlu [27].)

**Case II:** When  $\rho \neq 0$

Taking  $\zeta = 1$ , Eqn. (8) reduces to

$$p = \rho(\text{Stiff fluid}) \tag{26}$$

Use of Eqn. (13) (without loss of generality the constant of integration is considered as zero) and (26) in Eqn. (16), we get

$$\rho = \frac{9\eta\alpha_4}{4} \tag{27}$$

Also by using Eqn. (13) in Eqn. (12), we obtain

$$\rho = \frac{3\alpha_{44}}{16\pi\kappa} \tag{28}$$

Comparing Eqns. (27) and (28), we find

$$12\pi\kappa\eta = \frac{\alpha_{44}}{\alpha_4} \tag{29}$$

Integrating Eqn. (29), we get

$$\ln(\alpha_4) = 12\pi\kappa \int \eta dt + b_1 \tag{30}$$

Where,  $b_1$  is the constant of integration.

To avoid complexity of the problem and without loss of generality, we take  $b_1 = 0$ .

Thus, Eqn. (30) can be expressed as

$$\alpha_4 = e^{12\pi\kappa \int \eta dt} = e^{f(t)}, (\text{say}) \tag{31}$$

Where,  $f(t) = 12\pi\kappa \int \eta dt$ .

Again integrating Eqn. (31), we obtain

$$\alpha = \int e^{f(t)} dt + b_2 \tag{32}$$

Where,  $b_2$  is the constant of integration.

To avoid complexity and without loss of generality, we take  $b_2 = 0$ .

Thus Eqn. (32) with the help of Eqn. (13) can be expressed as

$$\alpha = \beta = \delta = \int e^{f(t)} dt \tag{33}$$

Applying (31) in (28) and (29), we get

$$p = \rho = \frac{3}{16\pi\kappa} e^{f(t)} \cdot f'(t) \tag{34}$$

and

$$\eta = \frac{1}{12\pi\kappa} f'(t) \tag{35}$$

The use of Eqn. (34) and Eqn. (14) yields

$$\bar{p} = -\frac{1}{16\pi\kappa} e^{f(t)}.f'(t) \tag{36}$$

Hence in view of Eqn. (33) the metric in Eqn. (4) takes the form

$$ds^2 = -dt^2 + e^{2\int e^{f(t)}dt} [dx^2 + e^{2x} dy^2 + dz^2] \tag{37}$$

Since the field equations are highly non-linear and undetermined, we consider following particular cases to get solutions.

**Subcase I:** Let  $f(t) = t$   
Thus, from Eqns. (33) to (36), we obtain

$$\begin{aligned} \alpha &= \beta = \delta = e^t, \\ p &= \rho = \frac{3}{16\pi\kappa} e^t, \\ \eta &= \frac{1}{12\pi\kappa} \end{aligned}$$

and

$$\bar{p} = -\frac{1}{16\pi\kappa} e^t$$

In this case, Eqn. (37) reduces to stiff fluid filled universe model and the equation is given by

$$ds^2 = -dt^2 + e^{2e^t} [dx^2 + e^{2x} dy^2 + dz^2] \tag{38}$$

**Subcase II:** Let  $f(t) = lnt$   
As in Subcase-I, we get

$$\begin{aligned} \alpha &= \beta = \delta = \frac{t^2}{2}, \\ p &= \rho = \frac{3}{16\pi\kappa} t, \\ \eta &= \frac{1}{12\pi\kappa} \frac{1}{t} \end{aligned}$$

and

$$\bar{p} = -\frac{1}{16\pi\kappa}$$

In this case, Eqn. (37) reduces to static universe model and the equation is described by

$$ds^2 = -dt^2 + e^{t^2} [dx^2 + e^{2x} dy^2 + dz^2] \tag{39}$$

Similarly, one can find the model for  $\rho = 3p$  by taking  $\zeta = \frac{1}{3}$  in Eqn. (8).

#### 4. Conclusion

We have considered the Bianchi type-III metric in bimetric theory when the source of gravitational field is viscous fluid. It is found that Bianchi type-III cosmological model does not exist, hence singularity free vacuum model is constructed. However, the stiff fluid model and the static models of the universe are found as special cases.

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#### References

- [1] N. Rosen, *Gen. Rel. Grav.* **4**, 435 (1973).
- [2] H. Yilmaz, *Gen. Rel. Grav.* **6**, 269 (1975).
- [3] M. Israelit, *Gen. Rel. Grav.* **11**, 25 (1979).
- [4] M. Israelit, *Gen. Rel. Grav.* **13**, 681 (1981).
- [5] D. R. K. Reddy and R. Venkateswarlu, *Astrophys. Space Sci.* **158**, 169 (1989).
- [6] G. Mohanty and P. K. Sahoo, *Czech. J. Phys.* **52**, 1041 (2002).
- [7] G. Mohanty and P. K. Sahoo, *Comm. Phys.* **13(3)**, 137 (2003).
- [8] D. R. K. Reddy, K. S. Adhav and S. D. Katore, *Astrophys. Space Sci.* **301**, 149 (2006).
- [9] D. R. K. Reddy, R. L. Naidu and V. U. M. Rao, *Int. J. Theor. Phys.* **47**, 1594 (2008).
- [10] P. K. Sahoo, *Int. J. Theor. Phys.* **47**, 3029 (2008).
- [11] P. K. Sahoo, *Int. J. Theor. Phys.* **48**, 2022 (2009).
- [12] P. K. Sahoo, *Int. J. Theor. Phys.* **49**, 25 (2010).
- [13] P. K. Sahoo and B. Mishra, *Int. J. of Pure and Appl. Math.* **82(1)**, 87 (2013).
- [14] P. K. Sahoo and B. Mishra, *J. Theor. and Appl. Phys.* **7**, 36 (2013).
- [15] S. Weinberg, *J. Astrophys.* **19**, 175 (1971).
- [16] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).
- [17] O. Gron, *Astrophys. Space Sci.* **173**, 213 (1990).
- [18] G. L. K. Murphy, *Phys. Rev.* **D8**, 4231 (1973).
- [19] R. Bali and R. D. Upadhaya, *Astrophys. Space Sci.* **283**, 1997 (2003).

- [20] X. X. Wang, Chin. Phys. Lett. **20**, 615 (2003).
- [21] X. X. Wang, Chin. Phys. Lett. **21**, 1205 (2004).
- [22] X. X. Wang, Chin. Phys. Lett. **22**, 29 (2005).
- [23] X. X. Wang, Chin. Phys. Lett. **23**, 1702 (2006).
- [24] M. K. Yadav, A. Rai and A. Pradhan, Int. J. Theor. Phys. **46**, 2677 (2007).
- [25] V. U. M. Rao and K. V. S. Sireesha, Int. J. Theor. Phys. **51**, 3013 (2012).
- [26] V. U. M. Rao and K. V. S. Sireesha, Eur. Phys. J. Plus. **127**, 49 (2012).
- [27] D. R. K. Reddy and Rao Venkateswara, Astrophys. Space Sci. **257**, 293 (1998).
- [28] V. U. M. Rao, D. Neelima and P. Suneetha, The African Rev. of Phys. **8**, 8 (2013).

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